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Non-monotone wedge Trust Region Method for Unconstrained Optimization

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Abstract: We consider non-monotone wedge trust region method for derivative-free optimization. Throughout our knowledge, this is the first investigation paper which combining non-monotone strategy into wedge trust region methods. The computational results proved the efficiency of the new composite strategy.
Keywords: wedge trust region, non-monotone technique, unconstrained optimization, derivative free optimization

INTRODUCTION

In this paper, we limit our discussion to the unconstrained optimization problem:

$$min\ f(x),\quad x \in R^n \tag{1}$$

where the objective function $f(x)$ is continuous and its derivatives are not available. There are several effective techniques solving this kind of problems, such as patter search methods, simplex methods, interpolation model [1, 2], and so on.

We consider trust region methods which are famous for having global convergence quality and some other good properties [3, 4]. The traditional trust region methods obtain a trial step by solving the following quadric model m_k,

$$m_k\left(x_k+s\right)=f\left(x_k\right)+g_k^T s+\frac{1}{2}s^T G_k s, \tag{2}$$

where the vector $g_k \in R^n$ and the $n \times n$ symmetric matrix G_k must be determined so that the model interpolates f at a set of sample points, as the following

$$m\left(x_k\right)=f\left(x_k\right), m_k(y^l)=f(y^l), l=1,2,\cdots,m, \tag{3}$$

where $Y_k=\left\{y^1,y^2,\cdots,y^m\right\}\cup\left\{x_k\right\}$ is the interpolation points set.

In order to insure the uniqueness and existence of the quadratic model, the parameter m should be chosen as $m=(n+1)(n+2)/2-1$ and the interpolation points set should be poised [5, 6, 7, 8] for the well defined quadratic models. When the model m_k is determined by these conditions, the interpolation set Y_k is nonsingular.

Let's suppose that we start the current iteration with a nonsingular set of sample points Y_k. Before computing a new trial point using the model m_k, we can figure out $y^{l_{out}}$ which is the farthest satellite from current iterate x_k. It can insure virtue of the models. In fact, the wedge trust region method is for computing a trial step s_k by approximately solving

$$\min_s m_k(x_k+s) \tag{4}$$

$$s.t.\|s\| \le \Delta_k \tag{5}$$

$$s \notin W_k, \tag{6}$$

where W_k is a set [9, 10, 11] which contains the "taboo region" area, and the purpose is to avoid the new point falling into it, and this trail step s_k is calculated by the method introduced in [8]. This idea is very promising.

Unfortunately, we could not find the optimal point rapidly. In the iteration, we must choose the relatively good point for the next iteration point.

Recently, non-monotone techniques are widely used in the trust region methods. Due to the high efficiency of non-monotone techniques, many authors are interested in working on the non-monotone techniques for solving optimization problems [12, 13]. the traditional non-monotone trust region method adoptions the ratio

$$\bar{r}_k = \frac{f_l(x_k) - f(x_k + s_k)}{m(0) - m_k(s_k)} \tag{7}$$

to measure the degree of this model and adjust the trust region radius. Actually, it may not be a true reflection of the current model fitting the objective function. This paper improved the algorithm of Deng [14] and applied it to the wedge trust region algorithm. We may use the ratio (7) to update the trust region radius

$$\hat{r}_k = \frac{D_k - f(x_k + s_k)}{m(0) - m_k(s_k)}$$

as a accepted rule of the tentative point.

The rest of this paper is organized as follow. In section 2, a new non-monotone wedge trust region algorithm will be established, and the algorithm analysis is explained. Numerical results are proved in section 3 which is showed that the new method is quite effective for unconstrained optimization problems. Finally, we give some conclusions in section 4.

A non-monotone wedge trust region algorithm

Step 0 Set the trial parameters, an initial trust region radius $\Delta_k > 0$, and an initial guess x_0. The interpolation set $Y_K = x_k \cup Y$, $Y = \{y^1, y^2, \cdots, y^m\}$ (we assume that $f(x_k) \le f(y) \forall y \in Y$).

Step 1 According to the current iteration point x_k, compute

$$y^{l_{out}} = \arg\max_{y \in Y} \|y - x_k\|.$$

Step 2 Construction quadratic model m_k and define the wedge W_k.

Step 3 Solve the sub-problem (2) and compute the trial step s_k, and calculate

$$r_k = \frac{Ared(d_k)}{\Pr ed(d_k)} = \frac{f(x_k) - f(x_k + s_k)}{m(0) - m_k(s_k)}, \quad \hat{r}_k = \frac{D_k - f(x_k + s_k)}{m(0) - m_k(s_k)}.$$

Step 4 Update the trust region radius Δ_k with the following Algorithm analysis.

Step 5 Update the interpolation set and the iteration point, if it is a successful iteration, that is $\hat{r}_k > \alpha_1$, then $x_{k+1} = x_k + s$, $Y = \{x_k\} \cup y / \{y^{l_{out}}\}$.

Else it is a unsuccessful iteration, that is $\hat{r}_k < \alpha_1$, then $x_{k+1} = x_k$,

$$Y = \begin{cases} \{x_k + s\} \cup y / \{y^{l_{out}}\}, if \ \|y^{l_{out}} - x_k\| \ge \|(x_k + s) - x_k\| \\ Y, \text{otherwise} \end{cases}.$$

Step 6 $k = k + 1$, go to step 1.

Algorithm analysis: In the traditional trust region algorithm, when the function value increases, the trust region radius must be reduced. But this situation is different with the case of non-monotone method when the function value could be allowed to increase. Therefore we can adopt some different parameters involved in updating the radius. Actually, The formulas for updating the trust region radius are the following:

$$\Delta_{k+1} = \begin{cases} \beta_4 \|s_k\|, r_1 < \alpha_1 且 \hat{r}_k > \alpha_1; \\ \beta_1 \|s_k\|, r_1 < \alpha_1 且 \hat{r}_k \leq \alpha_1; \\ \Delta_k, \alpha_1 \leq r_k < \alpha_2; \\ \beta_2 \Delta_k, \alpha_2 \leq r_k \leq \alpha_3; \\ \beta_3 \Delta_k, r_k > \alpha_3. \end{cases}.$$

This method still reduces the trust region radius when the function value decreases. In the numerical experiment, the parameters are constructed as follows, $\alpha_1 = 0.01$, $\alpha_2 = 0.95$, $\alpha_3 = 1.05$, $\beta_1 = 0.5$, $\beta_2 = 2$, $\beta_3 = 1.01$, $\beta_4 = 0.8$. In addition, the algorithm is different from the original one in [12]. We choose

$$D_k = \begin{cases} f(x_k) & k = 1 \\ \eta_k D_{k-1} + (1-\eta_k) f(x_k) & k \geq 2 \end{cases} \quad (8)$$

to replace the $f_{l(k)}$, and the parameter

$$\eta_k = \frac{f(x_k)}{f(x_k) - D_{k-1}}.$$

NUMERICAL RESULTS

Actually, this experiment is based on both quadratic model's traditional wedge trust region and non-monotone accepting criterion. The idea of solving the sub-problem is described carefully in Moré and Sorensen [8]. And the basic original program for wedge trust region algorithm is in [14]. We write the new algorithm by MATLAB. In this work, the traditional wedge trust region algorithm alg2 [9] based on quadratic model and the new algorithm are compared mainly according to number of function evaluations. Specially, we select 28 trial problems, which come from the CUTE [16]. Because the sub-program needs larger function calculation and the convergence are relatively slow, we only solve the right amount of dimension test questions. In the following table, the name of 28 test questions and results are given. We define n is the dimension of the objective function, and nf is the calculative times of an experimental function value.

Finally, f is the optimal point and the wed act represents the number of wedge constraints.

Table-1: Comparison non-monotone wedge trust region algorithm with wedge trust region based on quadratic model

n	p	nf		f		wed act	
		new	alg2	new	alg2	new	alg2
15	ARWHHEAD	482	479	8.44E-15	1.29E-14	25	29
10	BDQRTIC	376	440	1.83E+01	1.83E+01	25	28
6	BIGGS6	305	345	2.07E-06	3.74E-06	23	16
10	BROWNAL	471	512	3.65E-29	1.82E-29	38	43
2	BROWNBS	50	58	9.86E+11	9.82E+11	8	12
10	BRYBND	245	251	2.22E-28	5.25E-29	28	27
10	CRAGGLVY	690	700	1.89E+00	1.89E+00	26	35
2	CUBT	132	134	4.44E-31	4.93E-32	18	26
2	DENSCHNA	45	45	5.88E-37	4.72E-33	15	13
15	DIXMAANC	668	691	1.00E+00	1.00E+00	33	26
15	DIXMAANK	1070	1087	1.00E+00	1.00E+00	35	27
15	DIXON3DQ	94	99	1.68E-27	1.26E-27	8	8
6	ENGVAL1	37	43	0.00E+00	0.00E+00	13	14
2	ENGVAL2	102	105	2.03E-28	7.10E-29	24	21
5	GENHUMPS	194	270	3.11E-44	4.87E-42	24	28
3	GROWTHLS	91	74	1.22E+01	1.25E+01	6	7
2	HAIRY	58	89	2.00E+01	2.00E+01	58	89
6	HEART6LS	2350	3261	3.51E-01	3.06E-01	67	72
2	HUMPS	124	200	0.00E+00	1.30E-58	66	19
3	HATFLDE	53	118	4.58E-07	4.52E-07	14	16
3	HATFLDD	130	98	6.62E-08	3.28E-07	35	14
4	KOWOSB	130	158	3.08E-04	3.08E-04	28	31
5	OSBORNEA	162	197	1.51E-04	7.60E-05	10	17
11	OSBORNEB	1183	1261	4.01E-02	4.01E-02	41	36
3	PFIT1LS	260	260	2.01E+02	1.99E+02	14	18
2	SISSER	241	294	4.61E-57	5.54E-67	31	22
4	WOODS	351	407	4.94E-30	3.72E-30	26	23
10	VARDIM	5516	6494	6.35E+02	6.35E+02	30	29

According to the experiment, we can see that there are 24 problems of new algorithm performing better than the typical wedge trust region methods considering the number of function value's calculation, and the new algorithm reduced the time of computing the function value. At the same time, we get 7 problems which improved the precision of the optimal solution considerably. In the midst of all the questions there are 9 functions whose optimal solution is uniform, all of them being solved use less time of calculations than the traditional wedge trust region methods. The experimental results show that our new algorithm is effective.

CONCLUSIONS

In this paper, we propose a non-monotone wedge trust region method for unconstrained optimization without derivatives. The sub-problem incorporates more information which is useful to the algorithm. However, in the process of numerical experiments, we don't have comprehensively considering the influence of parameters on the efficiency of algorithm. In the near future, we would like to research the performance of the algorithm under different parameter condition.

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