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On Interesting Triple Integer Sequences

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Abstract: We search for three non-zero distinct integers such that each of the triple $(x-y, z, x+y)$ forms Harmonic progression. A few interesting properties among the solutions are also presented.
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INTRODUCTION

Number theory, called the Queen of Mathematics, is a broad and diverse part of Mathematics that developed from the study of the integers. The foundations for Number theory as a discipline were laid by the Greek mathematician Pythagoras and his disciples (known as Pythagoreans). One of the oldest branches of mathematics itself, is the Diophantine equations since its origins can be found in texts of the ancient Babylonians, Chinese, Egyptians, Greeks and so on[7-8]. Diophantine problems were first introduced by Diophantus of Alexandria who studied this topic in the third century AD and he was one of the first Mathematicians to introduce symbolism to Algebra. The theory of Diophantine equations is a treasure house in which the search for many hidden relation and properties among numbers form a treasure hunt. In fact, Diophantine problems dominated most of the celebrated unsolved mathematical problems. Certain Diophantine problems come from physical problems or from immediate Mathematical generalizations and others come from geometry in a variety of ways. Certain Diophantine problems are neither trivial nor difficult to analyze [1-6]. In this context one may refer [9, 10].

In this communication, we search for three non-zero distinct integers such that each of the triple $(x-y, z, x+y)$ forms Harmonic progression. A few interesting properties among the solutions are also presented.

METHOD OF ANALYSIS:

Let x, y, z be three non-zero distinct integers such that $(x-y, z, x+y)$ (*)
forms a harmonic progression (H.P).

By the definition of H.P, the above problem is equivalent to solving the Diophantine equation

$$x^2 - y^2 = xz \tag{1}$$

which is written as,

$$x^2 - xz - y^2 = 0 \tag{2}$$

Solving (2) through different approaches for finding x, y, z we get many triples of integers $(x-y, z, x+y)$ such that each triple forms a harmonic progression (H.P).

METHOD 1:

The substitution $y = kx, k > 1$ in (2) gives $z = (1-k^2)x$

Thus the triple $((1-k)x, (1-k^2)x, (1+k)x)$ forms harmonic progression (H.P).

METHOD 2:

The substitution $x = ky, k > 1$ in (2) gives $z = (k^2 - 1)\alpha, y = k\alpha$

Thus the triple $((k^2 - k)\alpha, (k^2 - 1)\alpha, (k^2 + k)\alpha)$ forms harmonic progression (H.P).

METHOD 3:

Treating (2) as quadratic in x and solving for x, we get

$$x = \frac{1}{2}[z \pm \sqrt{z^2 + 4y^2}] \quad (3)$$

consider $\alpha^2 = z^2 + 4y^2$ (4)

which is well known Pythagorean equation satisfied by

$$y = rs, z = r^2 - s^2, \alpha^2 = r^2 + s^2; r > s > 0 \quad (5)$$

Substituting (5) in (3), we get

$$x = r^2, -s^2 \quad (6)$$

Thus, in the view of (5) and (6), the non-zero distinct integer values of x, y, z satisfying (1) are given by

$$x = r^2, y = rs, z = r^2 - s^2$$

and

$$x = -s^2, y = rs, z = r^2 - s^2$$

In view of (*), we get the following triples forming H.P

$$(r^2 - rs, r^2 - s^2, r^2 + rs)$$

and

$$(-s^2 - rs, r^2 - s^2, r^2 + rs)$$

A few numerical examples are as follows:

r	s	TRIPLES
2	1	(2,3,6), (-3,3,1)
3	2	(3,5,15), (-10,5,2)
4	3	(4,7,28), (-21,7,3)
5	4	(5,9,45), (-36,9,4)
6	5	(6,11,66), (-55,11,5)

However, note that, (4) is also satisfied by,

$$2y = r^2 - s^2, z = 2rs, \alpha = r^2 + s^2; r > s > 0 \quad (7)$$

As our interest is on finding integer solutions, we choose r and s suitably so that the values of y and z are in integers.

Replacing r and s by 2R and 2S in (7)

$$y = 2(R^2 - S^2), z = 8RS, \alpha = 4(R^2 + S^2); R > S > 0 \quad (8)$$

Substituting the above values in (3),we get two sets of integer values of x, y, z satisfying (1) to be

$$x = 4RS + 2R^2 + 2S^2, y = 2(R^2 - S^2), z = 8RS$$

and

$$x = 4RS - 2R^2 - 2S^2, y = 2(R^2 - S^2), z = 8RS$$

Thus, the triples representing H.P are

$$(4RS + 4S^2, 8RS, 4RS + 4R^2)$$

and

$$(4RS - 4R^2, 8RS, 4RS - 4S^2)$$

A few numerical examples are as follows:

R	S	TRIPLES
2	1	(12,16,24), (-8,16,4)
3	2	(40,48,60), (-12,48,8)
4	3	(84,96,112), (-16,96,12)
5	4	(144,160,180), (-20,160,16)
6	5	(220,240,264), (-24,240,20)

Each of the following triples forms an Arithmetic progression:

- $(2x(x-y), xz, 2y(x-y))$
- $(2x^2 + x - y, xz, y - x - 2y^2)$
- $((2x^2 + x - y)z, xz^2, (y - x - 2y^2)z)$
- $(2x^2 + x, xz, -x - 2y^2)$
- $(2x^2 z, xz^2 - 2y^2 z)$
- $(2z(x^2 + x - y), xz^2, 2z(y - x - y^2))$
- $(2(x^2 + x - y), xz, 2(y - x - y^2))$

CONCLUSION

In this communication, we have exhibited different triples each forming a Harmonic progression. To conclude, one may search for other choices of triples forming Harmonic progression along with their corresponding properties

REFERENCES

1. Weil, A. (1987). Number Theory: An Approach through History, From Hammurapito to Legendre, Birkahsuser, Boston.
2. Datta, B., & Singh, A. N. (1935). History of Hindu mathematics.
1. Boyer, C., B. (1968). A History of mathematics", *John Wiley& sons Inc*., NewYork.
2. Dickson, L., E. (1952). History of Theory of Numbers", Vol. 11, Chelsea Publishing Company, New York.
3. Davenport, H. (1999). The higher Arithmetic: An Introduction to the Theory of Numbers" (7th ed.) *Cambridge University Press.*
4. Stilwell, J. (2004). Mathematics and its History", Springer Verlag, New York.
5. Matteson, J., M., D. (1888). A Collection of Diophantine problems with solutions", *Washington, Artemas Martin.*
6. Andrica, T., D. (2002). An Introduction to Diophantine equations" *GIL Publication House.*
7. Gopalan, M., A, Vidhyalakshmi, S., Thiruniraiselvi, N. (2015). On Two Interesting Diophantine Problems", *Impact Journal of Science and Technology*, Vol-9, No-3,51-55
8. M.A. Gopalan, M., A., Vidhyalakshmi, S., Kavitha, A. (2015). An application of second oder Ramanujan Number, *Jct Journal*, Vl-4, Issue-8, 40-41.
9. Gopalan, M.A.,Vidhyalakshmi, S. & Thiruniraiselvi N.(2015). On Two Interesting Diophantine Problems. *Impact Journal of Science and Technology*, Vol-9, No-3,51-55.
10. Gopalan, M.A, Vidhyalakshmi, S., & Kavitha, A. (2015). An application of second order Ramanujan Number. *Jct Journal*, Vl-4, Issue-8, 40-41.