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Implementation of optimizing slice interpolation in medical image using finite difference method

Nasim Khoonkari[1], Alireza Fakharzadeh Jahromi
[1]Department of Mathematics, Shiraz University of Technology, Fars, Shiraz, Modarres Boulevard, Iran

***Corresponding Author:**
Nasim Khoonkari
Email: n.khoonkari@gmail.com

Abstract: Slice interpolation in medical images is one of the important concepts in medical image processing in which many attempts have been made to improve it. In general, it is divided into two groups that are intensity-based interpolation and object-based interpolation. This article is an overview of object-based interpolation, based on image registration. Due to the various types of image registration and curvature registration as the latest one, interpolation is done on the basis of this type of registration. According to this, there would be an optimization problem which is solved by the use of Euler-Lagrange. In this regard, an optimization problem converted into a partial differential equation that is solved by using finite difference method. Then, by applying the solution an optimal displacement vector for moving the image is obtained.
Keywords: Finite Difference method, Image Registration, Medical Image Processing, Interpolation, Optimization.

INTRODUCTION

In modern imaging method (CT, MRI, etc.) a sequence of two dimensional images created, that can be used to provide and build three dimensional models. Because normally, the resolution is not the same in each of the three dimensions and significantly in z direction, it is less than x and y directions, in this regard, Interpolation algorithm that can strengthen the third dimension resolution, in a way that it become more symmetrical, are much needed.

In general, slice interpolation methods, which means finding one or more than one slice between two subsequent slices, can divide into two general categories[1,3]:

1. Intensity-based interpolation.
2. Object-based interpolation.

In the first, although it is simple and has low computational complexity, but these methods suffer from unrealistic and undesirable visual yield, while the second interpolation method leads to more accurate results. To reduce blurring of the edges and achieve better results, other methods of interpolation were also examined, that the most important one of them is the registration-based interpolation[1,3].

Image registration is a technique in image processing to align the multiple scenes in order to achieve a unified image. The advantage of this method is in overcoming issues such as rotation, scaling and offset in overlapping images. In general, two parametric and nonparametric registrations have been proposed till now[2]. These kind of methods established by two assumptions:

1. Subsequent slices have similar anatomical features.
2. Registration method is capable to find a map to match similar features in images.

Changes in any of these assumptions create false corresponding mapping, which leads to an undesirable interpolation.

Recent activities in the field of registration have been conducted by Xu et al. [4], that is a multipurpose registration method for slice interpolation of images. The other method for slice interpolation is curvature registration, which is proposed by Fischer B. and Modersitzki J. [1] and on 2014 Baghaie A. and Yu Z. implemented it for interpolating medical images that has a pleasant results [3]. In this article, the use of finite difference in solving the following optimization problem of slice interpolation based on registration will be noted and also implemented.

Image registration and Interpolation

As mentioned, the aim of slice interpolation in medical images is to create a three-dimensional image using two-dimensional slices that are generated by imaging devices. For this purpose, first the image can be defined as a function as follows:

Definition 2.1 Let $d \in \mathbb{N}$. A function $b : \mathbb{R}^d \to \mathbb{R}$ is called a d-dimensional image, if

1. Function b is compactly supported,
2. $0 \le b(x) < \infty$ for all $x \in \mathbb{R}^d$,
3. $\int_{\mathbb{R}^d} b(x)^k dx$ is finite, for $k > 0$.

Suppose R and T are two images, the aim of image registration is to find a local or global transformation from T to R such that T matches R. Also suppose that $R, T : \Omega \to \mathbb{R}$ are d-dimensional images, such that $\Omega := [0,1]^d$ and for particular $x \in \Omega$, T(x) is the gray value of spatial point x. Therefore, the aim of registration is to find a displacement field $u : \mathbb{R}^d \to \mathbb{R}$ such that T(x-u(x))=R(x), or in a way that T(x) become similar to R(x).

Obtaining suitable and optimal u=(u_1 u_2 … u_d) can be done by doing optimization. Indeed image registration can depend on landmarks of the image or it can be obtained directly from the gray values of the images. In this case, the second one will be used, because it is more flexible and also it is difficult to find the landmarks automatically. Image registration formulated as variational formulas, so the energy function can be defined as follows:

$$E(u) = D[R,T,u] + \alpha S[u];$$

in this formula D states distance measure, S defines the smoothing of u and α is the balance parameter between these two terms. In this function u must be found in a way that this function become minimized. D can be computed in several different ways and in this case it computed by sum of square difference which is defined as follows:

Definition 2.2 For $d \in \mathbb{N}$ and $R, T \in \mathrm{Img}(d)$, The sum of square differences (SSD) distance measure D^{SSD} between R, T is defined by $D^{SSD} : \mathrm{Img}(d)^2 \to \mathbb{R}$ as follows:

$$D^{SSD}[R,T] := \frac{1}{2}\|T - R\|_{L_2}^2 = \frac{1}{2}\int_{\mathbb{R}^d} (T(x) - R(x))^2 dx .$$

According to Fischer B. and Modersitzki J. [1] there are several ways for computing S, but as mentioned before in this article S obtained from curvature registration formula as follows [1,2]:

$$S(u) = \frac{1}{2}\sum_{l=1}^{d}\int_{\Omega} (\Delta u_l)^2 dx .$$

In this formula Δ state for curvature operator which is defined as laplacian in mathematic that known as $\nabla.\nabla$.

Optimization method for image registration based on finite difference

The goal is to interpolate a slice between two consecutive slices with similar anatomical characteristics and with the use of curvature registration. In this case the energy function should be minimized with respect to u in order to find the in-between slice with a simple average of corresponding points of the registered images. Therefore the objective function of this optimization problem demonstrated as follows:

$$\min_u E(u) = D[R,T,u] + \alpha S(u) .$$

The following theorems can be used to solve the optimization problem with use of finite difference [2].

Theorem 2.3 For $R, \mathrm{T} \in \mathrm{Img(d)}$, $T \in C^2(\mathbb{R}^d)$, $u, v : \mathbb{R}^d \to \mathbb{R}^d$, $\Omega := [0,1]^d$. The Gateaux derivative [5] of D[R,T;u],

$$D[R,T;u] := \frac{1}{2}\|T_u - R\|_{L_2(\Omega)} ,$$

with respect to v is given by

$$dD[R,T;u;v] = -\int_{\mathbb{R}^d} < f(x,u(x)), v(x) >_{\mathbb{R}^d} dx \ ,$$

where $f : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}^d$

$$f(x,u(x)) := (R(x) - T_u(x)) \nabla T_u(x) \ .$$

Theorem 2.4 The Euler-Lagrange equation for $E = D + \alpha S$, where D is sum of square difference and S in curvature registration is:

$$f(x,u(x)) + \alpha \Delta u(x) = 0, x \in \Omega,$$

$$\nabla u_l(x) = 0 \text{ for } x \in \partial\Omega, l = 1,2,...,d \ .$$

Where u is demonstrated by index l in order to show that the displacement must be computed in all dimensions of image that in this case the dimension is 2 because of the images are 2-dimensional.

Now for computing u we use finite difference method. First a time step τ and k>0 introduced and the uniformly spaced grid points are defined as follows:

$$x_{i_1,i_2,...,i_d} := (\frac{2i_1 - 1}{2n_1}, \frac{2i_2 - 1}{2n_2}, ..., \frac{2i_d - 1}{2n_d}) \in \Omega, i_l = 1,2,...,n_l, l = 1,2,...,d \ .$$

Then, grid points $n := n_1, n_2, ..., n_d$ from matrix $X \in \mathbb{R}^{n_1 \times n_2 \times ... \times n_d}$ obtained. Also, we have:

$$U^{(k)} := (U_1^{(k)}, U_2^{(k)}, ..., U_d^{(k)}) \ , \ U_l^{(k)} := u_l(X, k\tau) \ ,$$

$$F^{(k)} := (F_1^{(k)}, F_2^{(k)}, ..., F_d^{(k)}) \ , \ F_l^{(k)} := f_l(X, u(X, k\tau)) \ .$$

In this regard, the derivative of $u(X, k\tau)$ have been estimated by finite difference method as follows:

$$u(X, k\tau) \approx \frac{U_l^{(k+1)} - U_l^{(k)}}{\tau} \ .$$

Then the equation based on finite difference method is

$$\partial_t u^{k+1}(x,t) = f(x, u^k(x,t) + \alpha \Delta^2 [u^{k+1}](x,t), k > 0,$$

$$\frac{u^{k+1} - u^k}{\Delta t} = f(x, u^k(x,t) + \alpha \Delta^2 [u^{k+1}](x,t), k > 0,$$

Now by substituting τ instead of Δt , the equation obtained as

$$(I_n + \tau \alpha \Delta^2) U_l^{(k+1)} = U_l^{(k)} + \tau F_l^{(k)} .$$

Then u can be obtained from this equation and the registration can be done by T(x-u(x))=R(x). Interpolation can be done by the average of the gray values of corresponding pixels of registered image T and image R.

$$R = Interpolation(\frac{Tou(x) + R(x)}{2}) \ .$$

According to Baghaie et al. [3] for minimizing the displacement vector u , two subsequent slices R_1 and R_2 can be considered as two given slices, then by the use of following formula the inbetween slice can be obtaind by moving both of the given images:

$$R = Interpolation(\frac{R_1(x - u) + R_2(x + u)}{2}) \ .$$

Result and Disscution

This article is an overview on the use of image registration in 3-dimentional image interpolation, as well as showing the use of finite difference in solving the mentioned problem. Although Finite difference is a remarkable and accurate method, but it increase the iterations of the algorithm.

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