A unified approach for non-monotone Pattern search algorithm
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Abstract: In this paper, pattern search method for unconstrained optimization problems proposed with a new strategy of non-monotonic technique. Also the convergence of this algorithm will be analyzed, theoretically. This analysis shows the global convergence of this algorithm.

Keywords: Unconstrained optimization, pattern search, non-monotonic technique, convergence analysis.

INTRODUCTION
In this paper, we consider the unconstrained minimization problem as follows:

$$\text{Min } f(x), \quad x \in \mathbb{R}^n,$$

Where $$f : \mathbb{R}^n \to \mathbb{R}$$, and continuously differentiable and also there is not any information about the derivative of a function $$f(x)$$ or the information are unreliable. In this case, the optimization problem is on a derivative free function. Pattern search is among the methods that can be used for derivative free optimizations. Here, the direct search method can be applied to derivative free functions by using the values of only objective function and compare each trial solution with the best previous solution.

Although because of the shrink of available data, this method is difficult to be implemented and the convergence rate of it is slower than using derivative methods, but pattern search is a special kind of direct search method which was conducted by Hook and Javees [4] in 1961. Pattern search contains two kinds of move: 1. Pattern move, 2. Exploratory move. The first one provides actual minimizing of the objective function as well as the second one provides probable directions.

Pattern search method
The pattern can be defined by two components, a basis matrix and a generating matrix. The basis matrix can be any nonsingular matrix $$B \in \mathbb{R}^{n \times n}$$. The generating matrix is a matrix $$C_k \in \mathbb{Z}^{n/k \times n}$$, where $$P_k > n + 1$$. The generating matrix can be shown as follows:

$$C_k = [\Gamma_k | L_k | 0]$$

For our implementation we require that $$\Gamma_k \in M$$, and $$M$$ is a finite set of integral matrices with full row rank. We will see that $$\Gamma_k$$ must have at least $$n + 1$$ columns. The last column is a single column of zeros. For more information, check [1, 2, 3]. A Pattern $$P_k$$ is then defined by the columns of the matrix $$P_k = BC_k$$. The partition of the generating matrix $$C_k$$ to partition $$P_k$$ were used as follows:

$$P_k = BC_k = [B \Gamma_k | B L_k | 0].$$

Given $$\Delta_k \in \mathbb{R}^n, \Delta_k > 0$$, we define a trial step $$s^i_k$$ to be any vector of the form $$s^i_k = \Delta_k BC^i_k$$, where $$C^i_k$$ is a column of $$C_k$$ which $$BC^i_k$$ determines the direction of the step, while $$\Delta_k$$ serves as a step length parameter. A trial point as any point of the form $$x^i_k = x_k + s^i_k$$ at iteration $$k$$ were defined. The following algorithm states the pattern search method for unconstrained minimization which was presented in [5]:
Algorithm 1
Let $x_0 \in R_n$ and $\Delta_0 > 0$ be given.
For $k = 0, 1, 2, \cdots$
1) Compute $f(x_k)$.
2) Determine a step $s_k$ using an unconstrained exploratory moves algorithm.
3) If $f(x_k + s_k) < f(x_k)$, then $x_{k+1} = x_k + s_k$, otherwise $x_{k+1} = x_k$.
4) Update $C_k$ and $\Delta_k$.
Especially, $\Delta_k$ should be updated by the following standards.

Let $\tau \in Q$, $\tau > 1$, and $\{w_0, w_1, \ldots, w_n\} \subset Z$, $w_0 < 0$, and $w_i \geq 0, i = 1, \cdots, l$.

$$\min \{ f( x_k + \Delta_k p^i_k) , i = 1, 2, \ldots, 2n \} < R_k$$

Let $\theta = \tau^{-\alpha}$ and $\lambda \in \tau = \{ \tau \}$.

$$f(x_{k+1}) \equiv f(x_k + \Delta_k d_k) \leq \min \{ f(x_k + \Delta_k p^i_k) , i = 1, 2, \ldots, 2n \}$$

Let $f(x_0) > 0$, then $R_{k+1} = \Delta_0 \theta$.

Let $f(x_0) < 0$, then $R_{k+1} = \Delta_{k+1}$. The motivation behind this non-monotone term is that the best convergence results are obtained by stronger non-monotone strategy whenever iterates are close enough to that. However, it does not allow the iterates to act as non-monotonic iterate in the first iterations. Ataee Tarzanagh et.al [9] proposed a non-monotone term by replacing $R_k$ with $(1 + \Phi_k)R_k$ in the trust region ration, where

$$\Phi_k = \begin{cases} \eta_k, & \text{if } R_k > 0 \\ 0, & \text{if } R_k < 0 \end{cases}$$

And $\{\eta_k\}$ is a positive sequence satisfying the following relation:

$$\sum_{k=1}^{\infty} \eta_k \leq \eta < \infty$$
This replacement allows us to have values greater than $f_k$ in the first iterations of the algorithm.

Algorithm 2:
Non-monotone pattern search algorithm
Step1: $x_k, \Delta_k > 0, R_k, \theta \in (0,1)$
Step2: For $i = 1,...,r$ if $f(x_k + \Delta_k p_i^k) < R_k$ go to step 3
Step3: Set $\Delta_k = \theta \Delta_k$ and go to step 1
Step4: Choose $x_{k+1} = x_k + \Delta_k d_k, d_k \in \{p^1_k, ..., p^r_k\}$, such that $f(x_{k+1}) \leq \min \{f(x_k + \Delta_k p_i^k), i = 1,...,r\}$
Step5: Set $\Delta_{k+1} = \Delta_k$
Step6: Compute $R_{k+1}$

Convergence Analysis

Problem assumptions:
(A1) $f = \mathbb{R}^n \rightarrow \mathbb{R}$ continuously differentiable.
(A2) The level set $L_0 = \{x \in \mathbb{R}^n : f(x) \leq e^0 f(x_0)\}$ is bounded.

Preposition: If $\min \{f(x_k + \Delta_k p_i^k), i = 1,...,2n\} < R_k$, then $f(x_{k+1}) = f(x_k + \Delta_k d_k) \leq \min \{f(x_k + \Delta_k p_i^k), i = 1,...,2n\}$.

Lemma: Suppose that assumption (A2) holds and that, method has constructed an infinite sequence $\{x_k\}$. Then $\lim \inf x_k = 0$.

Proof: For a given $k$, let $I(k)$ be an index in the set $\{k, k-1,..., k - \min(k,q)\}$, such that $f(x_{I(k)}) < R_k$; then it follows that $f(x_{I(k+1)}) = R_{k+1} \leq R_k = f(x_{I(k)})$, and $f(x_{I(k)}) < R_{I(k+1)} = f(x_{I(I(k)-1)})$. In other words, $\{f(x_k)\}$ admits an infinite strictly monotone subsequence and then $\{x_k\}$ contains an infinite number of distinct points. We proceed now by contradiction, strictly following the proof of theorem 3.3 in [10]. Assuming that $0 < \Delta_{LB} \leq \Delta_k$, for some $\Delta_{LB}$ independent of $k$, it follows from

$$\Delta_k = \tau^k \Delta_0, r_k \in \mathbb{R}$$

$x_N = x_0 + (\beta^\tau x \alpha^\tau x_a) \Delta_0 B \sum_{k=0}^{N-1} z_k, z_k \in \mathbb{R}^n$

That all the iterates belong to finite set, thus leading to a contradiction.

Theorem: Suppose that the assumptions (A1) and (A2) hold and that sequence $\{x_k\}$ is generated by Algorithm 2. If there exist $c_1,c_2 > 0$ such that $\forall k$

$$c_1 \leq ||p_i^k|| \leq c_2, i = 1,...,r$$

Then $\lim_{k \rightarrow \infty} \|\nabla f(x_k)\| = 0$.

Proof: The proof is simply obtained by the Theorem 2.5 in [12] and Theorems 7 and 8 in [11].

CONCLUSION

In this paper, according to the novel researches on non-monotone techniques, we introduced a unified approach for non-monotone pattern search algorithms. Also, we proved that this method is convergence, according to theorem and lemma which we used.
REFERENCES

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