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A unified approach for non-monotone Pattern search algorithm

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Abstract: In this paper, pattern search method for unconstrained optimization problems proposed with a new strategy of non-monotonic technique. Also the convergence of this algorithm will be analyzed, theoretically. This analysis shows the global convergence of this algorithm.
Keywords: Unconstrained optimization, pattern search, non-monotonic technique, convergence analysis.

INTRODUCTION

In this paper, we consider the unconstrained minimization problem as follows:

$$\text{Min } f(x),\ x \in R^n,$$

Where $f = R^n \to R$, and continuously differentiable and also there is not any information about the derivative of a function $f(x)$ or the information are unreliable. In this case, the optimization problem is on a derivative free function. Pattern search is among the methods that can be used for derivative free optimizations. Here, the direct search method can be applied to derivative free functions by using the values of only objective function and compare each trial solution with the best previous solution.

Although because of the shrink of available data, this method is difficult to be implemented and the convergence rate of it is slower than using derivative methods, but pattern search is a special kind of direct search method which was conducted by Hook and Javees [4] in 1961. Pattern search contains two kinds of move: 1. Pattern move, 2. Exploratory move. The first one provides actual minimizing of the objective function as well as the second one provides probable directions.

Pattern search method

The pattern can be defined by two components, a basis matrix and a generating matrix. The basis matrix can be any nonsingular matrix $B \in R^{n\times n}$. The generating matrix is a matrix $C_k \in Z^{n\times p_k}$, where $P_k > n+1$. The generating matrix can be shown as follows:

$$C_k = [\Gamma_k \mid L_k \mid 0]$$

For our implementation we require that $\Gamma_k \in M$, and M is a finite set of integral matrices with full row rank. We will see that Γ_k must have at least $n+1$ columns. The last column is a single column of zeros. For more information, check [1,2,3]. A Pattern P_k is then defined by the columns of the matrix $P_k=BC_k$. The partition of the generating matrix C_k to partition P_k were used as follows:

$$P_k=BC_k=[B\Gamma_k \mid BL_k \mid 0].$$

Given $\Delta_k \in \square, \Delta_k > 0$, we define a trial step s_k^i to be any vector of the form $s_k^i = \Delta_k BC_k^i$, where C_k^i is a column of C_k which BC_k^i determines the direction of the step, while Δ_k serves as a step length parameter. A trial point as any point of the form $x_k^i = x_k + s_k^i$ at iteration k were defined. The following algorithm states the pattern search method for unconstrained minimization which was presented in [5]:

Algorithm 1

Let $x_0 \in R_n$ and $\Delta_0 > 0$ be given.
For $k = 0, 1, 2, \cdots$
1) Compute $f(x_k)$.
2) Determine a step s_k using an unconstrained exploratory moves algorithm.
3) If $f(x_k + s_k) < f(x_k)$, then $x_{k+1} = x_k + s_k$, otherwise $x_{k+1} = x_k$.
4) Update C_k and Δ_k.
Especially, Δ_k should be updated by the following standards.
Let $\tau \in Q$, $\tau > 1$, and $\{w_0, w_1, ..., w_n\} \subset Z$, $w_0 < 0$, and $w_i \geq 0, i = 1, \cdots, l$.

$$\min\{f(x_k + \Delta_k p_k^i), i = 1, ..., 2n\} < R_k$$

Let $\theta = \tau^{\omega 0}$ and $\lambda \in \wedge = \{\tau, \cdots, \tau^{wl}\}$

$$f(x_{k+1}) \equiv f(x_k + \Delta_k d_k) \leq \min\{f(x_k + \Delta_k p_k^i), i = 1, ..., 2n\}$$

a) If $f(x_k + s) \geq f(x_k)$, then $\Delta_{\kappa+1} = \Delta_\kappa \theta$.
b) If $f(x_k + s) < f(x_k)$, then $\Delta_{\kappa+1} = \Delta_\kappa \lambda_k$.
Obviously, the conditions on θ and $\wedge$ should ensure that $0 < \theta < 1$ and $\lambda_i \geq 1$ for all $\lambda_i \in \wedge$.

Classical pattern search methods try to satisfy the monotone decrease condition $f(x_{k+1}) < f(x_k)$, and then they use the reference value $R_k = f(x_k)$. Here, this requirement is relaxed, the objective function value is allowed to increase at some iterations and reduction is required only in the space of several iterations; this is the distinguishing feature of non-monotone algorithms, and corresponds to make use of a reference value $R_k \geq f(x_k)$, provided that $R_{k+1} \leq R_k$ for each k (at the first iteration, $R_0 = f(x_0)$ is commonly used.

The sampling about x_k in search of a new iteration $x_{k+1} = x_k + \Delta_k d_k$ with $d_k \in \{p_k^1, ..., p_k^r\}$, is named exploratory moves. Exploratory moves may satisfy two different conditions , a weak condition and a strong condition.

Non-monotone technique and our strategy

Non-monotone technique are used in the line search and trust region methods in recent decades. In 1982, Chamberlian et al. [6] proposed the first non-monotonic technique for constrained optimization which is called watching technique. According to the high efficiency of such techniques, many authors interested in working on these techniques for solving optimization problems.

There are various non-monotone algorithms, Ahookosh et.al [7] introduced a non-monotone term as follows to overcome suffering from drawbacks which was appearing in Grippo's techniques [8]. The non-monotone term is:

$$R_k = \varepsilon_k f_{l(k)} + (1 - \varepsilon_k) f_k$$

In which:

$$f_{l(k)} = \max_{0 \leq j \leq \min(k, M)} \{f(x_{k-j})\}$$

Where $f_k = f(x_k)$ and $\varepsilon_k \in [\varepsilon_{\min}, \varepsilon_{\max}] \subset [0,1]$. The motivation behind this non-monotone term is that the best convergence results are obtained by stronger non-monotone strategy whenever iterates are close enough to that. However, it does not allow the iterates to act as non-monotonic iterate in the first iterations. Ataee Tarzanagh et.al [9] proposed a non-monotone term by replacing R_k with $(1 + \Phi_k) R_k$ in the trust region ration, where

$$\Phi_k = \begin{cases} \eta_k, & if \quad R_k > 0 \\ 0, & if \quad R_k < 0 \end{cases}$$

And $\{\eta_k\}$ is a positive sequence satisfying the following relation:

$$\sum_{k=1}^{\infty} \eta_k \leq \eta < \infty$$

This replacement allows us to have values greater than f_k in the first iterations of the algorithm.

Algorithm 2:
Non-monotone pattern search algorithm

Step1: $x_k, \Delta_k > 0, R_k, \theta \in (0,1)$

Step2: For $i = 1,\ldots,\mathrm{r}$ if $f(x_k + \Delta_k p_k^i) < R_k$ go to step 3

Step3: Set $\Delta_k = \theta\Delta_k$ and go to step 1

Step4: Choose $x_{k+1} = x_k + \Delta_k d_k, d_k \in \{p_k^1,\ldots,p_k^r\}$, such that $f(x_{k+1}) \le \min\{f(x_k + \Delta_k p_k^i), i = 1,\ldots,r\}$

Step5: Set $\Delta_{k+1} = \Delta_k$

Step6: Compute R_{k+1}

Convergence Analysis

Problem assumptions:

(A1) $f = R^n \to R$ continuously differentiable**.**

(A2) The level set $L_0 = \{x \in R^n : f(x) \le e^{\eta} f(x_0)\}$ is bounded.

Preposition: If $\min\{f(x_k + \Delta_k p_k^i), i = 1,\ldots,2n\} < R_k$, then $f(x_{k+1}) \equiv f(x_k + \Delta_k d_k) \le \min\{f(x_k + \Delta_k p_k^i), i = 1,\ldots,2n\}$.

Lemma: Suppose that assumption (A2) holds and that, method has constructed an infinite sequence $\{x_k\}$. Then $\liminf_{x\to\infty} \Delta_k = 0$.

Proof: For a given k, let $l(k)$ be an index in the set $\{k, k-1,\ldots,k-\min(k,q)\}$, such that $f(x_{l(k)}) < R_k$; then it follows that $f(x_{l(\mathrm{k}+1)}) = R_{k+1} \le R_k = f(x_{l(k)})$, and $f(x_{l(k)}) < R_{l(k)-1} = f(x_{l(l(k)-1)})$. In other words, $\{f(x_k)\}$ admits an infinite strictly monotone subsequence and then $\{x_k\}$ contains an infinite number of distinct points. We proceed now by contradiction, strictly following the proof of theorem 3.3 in [10]. Assuming that $0 < \Delta_{LB} \le \Delta_k$, $\forall k$, for some Δ_{LB} independent of k, it follows from

$$\Delta_k = \tau^{rk}\Delta_0, r_k \in \mathbb{Z}$$

$$x_N = x_0 + (\beta^{r_{LB}}\alpha^{-r_{UB}})\Delta_0 B\sum_{k=0}^{N-1} z_k, z_k \in \mathbb{Z}^n$$

That all the iterates belong to finite set, thus leading to a contradiction.

Theorem: Suppose that the assumptions (A1) and (A2) hold and that sequence $\{x_k\}$ is generated by Algorithm 2. If there exist $c_1, c_2 > 0$ such that $\forall k$

$$c_1 \le \| p_k^i \| \le c_2, \; i = 1,\ldots,r$$

Then $\lim_{k\to\infty} \|\nabla f(x_k)\| = 0$.

Proof: The proof is simply obtained by the Theorem 2.5 in [12] and Theorems 7 and 8 in [11].

CONCLUSION

In this paper, according to the novel researches on non-monotone techniques, we introduced a unified approach for non-monotone pattern search algorithms. Also, we proved that this method is convergence, according to theorem and lemma which we used.

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