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Conformity Analysis Between Experimental and Theoretical Data in the Study Collisions of Relativistic Heavy Ion (The Case ϕ-Space)

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Abstract: We applied the wavelet methodology to study the results of the chaotic behavior of the production of particles in relativistic collisions of heavy ions. We use wavelet coherence to analyze the correspondence between theoretical and experimental data. We examined the 1-D phase space of variable (the case ϕ-space). We also compared the wavelet coherence values for η-space and ϕ-space. It was also shown that the values of the wavelet of coherence depend on the values of the parameters p and q. We discussed our new results for the comparison purpose and findings were in the good agreements.
Keywords: wavelet coherence, heavy ion, relativistic dynamics, phases space, normalized factorial moments, experimental and theoretical data, ϕ-space.

INTRODUCTION

Particle physics is one of the directions of modern physics, which is of great importance for studying the structure of matter. The basis of elementary particle physics is the study of the high nuclear energy of matter. The multiplicity of charged particles in high energy nucleus-nucleus interactions is an important parameter which indicates how many particles are produced in that interaction [1, 2].

The distribution of the multiplicity of the formed particles can be studied on the basis of an analysis of the spatial fluctuations of relativistic shower particles (E-by-E). This can be done by examining the time series of different collision rates of elementary particles [3]. In [4, 5] showed that the ultra-relativistic heavy-ion collisions provide a system in which the properties of hot, dense strongly interacting matter can be. Also have been numerous experimental results, which indicate that collisions of nucleus-nucleus interactions can not be completely understood in term of superposition of nucleon-nucleon scattering [6, 7, 8]. There are many studies where the experimental results have been compared with the data generated with the computer code FRITIOF based on Lund Monte Carlo Model for high energy nucleus-nucleus collisions [9, 10]. But in such experiments, the connection between the theoretical and experimental points of view is important. This will help to understand the complex moments that may arise. The theory predicts a phenomenon that can be verified experimentally, and experiments very often give new insights to unexpected results, which in turn leads to an improvement in the theoretical description.

Thus, we consider the relationship between experimental and theoretical data. However, such research is important to do in each case. In the present paper, we investigate the correspondence between theoretical and experimental data for the spatial fluctuations of relativistic shower particles arising in collisions of ^{28}Si+Em at energy14.6A GeV in 1-D phase space of X –variable (the case ϕ-space) [11]. The results are compared with the predictions of the model of ultrarelativistic quantum molecular dynamics (UrQMD) [12, 13]. For such an analysis, we will use the theory of wavelets. It is connected with the fact that chaotic behavior in relativistic heavy ion collisions can have a rather complex structure, contain local features of various shapes and times.

PHASE SPACES AND ANGULAR MEASUREMENTS

When the nuclei collide, the particles scatter at different angles. This makes it possible to measure angles in different phase spaces. These measurements are the basis for the study of chaotic behavior in relativistic heavy ion collisions. We consider two corners: space angle and azimuthal angle. These two corners form a 1-D phase space.

Since the direct measurement of the space angle is not possible, therefore knowing the projected angle (θ_P) and the dip angle (θ_d) of particular track of emission of a particle, one can easily, determines its value by the following relation [14]:

$$\theta_S = \cos^{-1}[\cos\theta_P \times \cos\theta_d]. \qquad (1)$$

This space is called pseudo-rapidity and is identified as a η-space.

In order to study the intermittency, multifractality, anisotropic flow and other related phenomena in relativistic nuclear collisions in two dimensions, the measurement of azimuthal angle is taken into account. The azimuthal angle, ϕ, is determined by the following relation [14]:

$$\phi = \cos^{-1}[\cos\theta_d \ \sin\theta_p \ / \sin\theta_S]. \qquad (2)$$

This space is called rapidity values and is identified as a ϕ-space. Thus, we can get different values for the normalized factorial moments.

In [15] we considered collisions of relativistic heavy ion for the case η-space. Here we will look at collisions of relativistic heavy ion for the case ϕ-spase.

MATHEMATICAL FORMALISM FOR CALCULATING NORMALIZED FACTORIAL MOMENTS

Analysis of chaotic behavior in relativistic heavy ion collisions is based on the calculation and study of normalized factorial moments. These normalized factorial moments relate to the chaotic nature of the system. At the same time, a new normalized moment related (C) to the chaotic nature of the system is defined in ϕ-spase as [16, 17]:

$$C_{p,q}(M) = <\phi_q^p(M)> = \frac{1}{N_{ev}} \sum_{e=1}^{N_{ev}} \phi_q^p(M), \qquad (3)$$

where, the usual meaning of various symbols / and or parameters are such as: p is any positive real number, it should not be negative, $F_q^e(M)$ may vanish for some events, if p is negative; ϕ – a normalized factorial moment in 1-D phase ϕ-space of $X(\phi)$ – variable ($X(\phi)$ – a variable that characterizes the number of relativistic heavy ion collisions); N_{ev} – is the number of events in a sample.

The event factorial moments, F_q^e, fluctuates from event-to-event, and the degree of fluctuation can be estimated from the probability distribution $P(F_q^e)$ over all events [11, 18]:

$$\phi_q(M) = \frac{F_q^e(M)}{< F_q^e(M) >}, \qquad (4)$$

$$F_q^{(e)} = \left[\frac{1}{M}\sum_{m=1}^{M} n_m(n_m - 1).......(n_m - q + 1)\right] \times \left(\frac{1}{M}\sum_{m=1}^{M} n_m\right)^{-q}, \qquad (5)$$

$$< F_q^e(M) > = \frac{1}{N_{ev}} \sum_{e=1}^{N_{ev}} F_q^e(M), \qquad (6)$$

where, M is the partition number in phase space, n_m is the number of shower tracks producing particles falling into the m^{th} bin and q = 2.3.4…. is the order of the moment; $F_q^e(M)$ represents the event factorial moment describing the spatial pattern of an event.

At the same time in order to perform a meaningful analysis of chaoticity, normalized cumulative variable ($X(\phi)$) were used to reduce the effect of non-uniformity in single charged particle distributions. The single charged particle density distribution is not flat in the analysis of the fluctuation in phase space variable. This non-uniformity of the particle spectra influences the scaling behaviour of scaled factorial moments. Bialas and Gazdzicki [18] proposed a method to construct a set of variables, which drastically reduces the distortion of intermittency due to the non-uniformity of single particle density distribution. According to them, the new scaled variable $X(\phi)$ is related to the single particle density distribution $\rho(\phi)$ by the following relation [11, 18]:

$$X(\phi) = \frac{\int_{\phi_1}^{\phi} \rho(\phi')\,d(\phi')}{\int_{\phi_1}^{\phi_2} \rho(\phi')\,d(\phi')} , \qquad (7)$$

where, $\rho(\phi) = (1/N)\,dn/d\phi$ is the single particle pseudo rapidity density distribution of the shower particles and ϕ_1 and ϕ_2 are the two extreme points in the distribution ρ(ϕ) (or $\phi_1 = \phi_{min}$, $\phi_2 = \phi_{max}$). Should also be noted that, in terms of new scaled variable ($X(\phi)$) the single particle density distribution is always uniform in between X = 0 and 1.

Various experimental efforts have established the existence of the empirical phenomenon of "intermittency" in multiparticle production using normalized scaled factorial moments. On the basis of bin averaging the normalized scaled factorial moments of the order of q is defined in vertical form as [11, 18]:

$$F_q^V(\delta\phi) = \frac{1}{M^d} \sum_{m=1}^{M^d} \frac{< n_m^q >}{< n_m >^q} \qquad (8)$$

and its horizontal form is defined as [11, 18]:

$$F_q^H(\delta\phi) = < \frac{1/M^d \sum_{m=1}^{M^d} n_m^q}{(< n > / M^d)^q} > , \qquad (9)$$

where, $n_m^q = n_m(n_m - 1).....(n_m - q + 1)$, and also bracket $< .... >$ of Eq. (8) indicates the average over all events in the whole data sample;

n_m is the number of relativistic charged particles in the m^{th} bin, m can take values from 1 to M represents the total multiplicity of charged shower particles in a particular event in the pseudo rapidity interval $\Delta\phi = M\delta\phi$ or $\delta\phi = \{X(\phi_{max}) - X(\phi_{min})\}/M$.

Thus, we can obtain a series of time series for normalized factorial moments $C_{p,q}(M)$ (both theoretical and experimental). To compare these series, we use the wavelet methodology – wavelet coherence.

WAVELET COHERENCE AS AN ANALYTICAL TOOL

Wavelet coherency simultaneously assess how the co-movement and causalities between two variables vary across different frequencies involved and change over time in a time-frequency window. Moreover, co-movement of the time series is observable among different time scales, which the standard approaches, failed to perform. It allows to calculate local correlation of two time series (x and y, where x are the theoretical values for $C_{p,q}(M)$, and y are the experimental values for $C_{p,q}(M)$) in a region of time-frequency. It uses the following formalized model: wavelet coherence as the squared absolute value of the smoothed cross wavelet spectra $D_{xy}(u,s)$, normalized by the product of the smoothed individual wavelet power spectra of each series [19, 20]:

$$Z^2(u,s) = \frac{\left|Q(s^{-1}D_{xy}(u,s))\right|}{Q(s^{-1}\left|D_x(u,s)\right|^2)Q(s^{-1}\left|D_y(u,s)\right|^2)}, \qquad (10)$$

where u – is a location parameter; s – is a scale parameter; Q – is a smoothing operator. To calculate the wavelet coherence for two series of data, we use the Morlet wavelet (D).

DATA

For the analysis we will use the data presented in [11].These data are presented in Fig. 1.

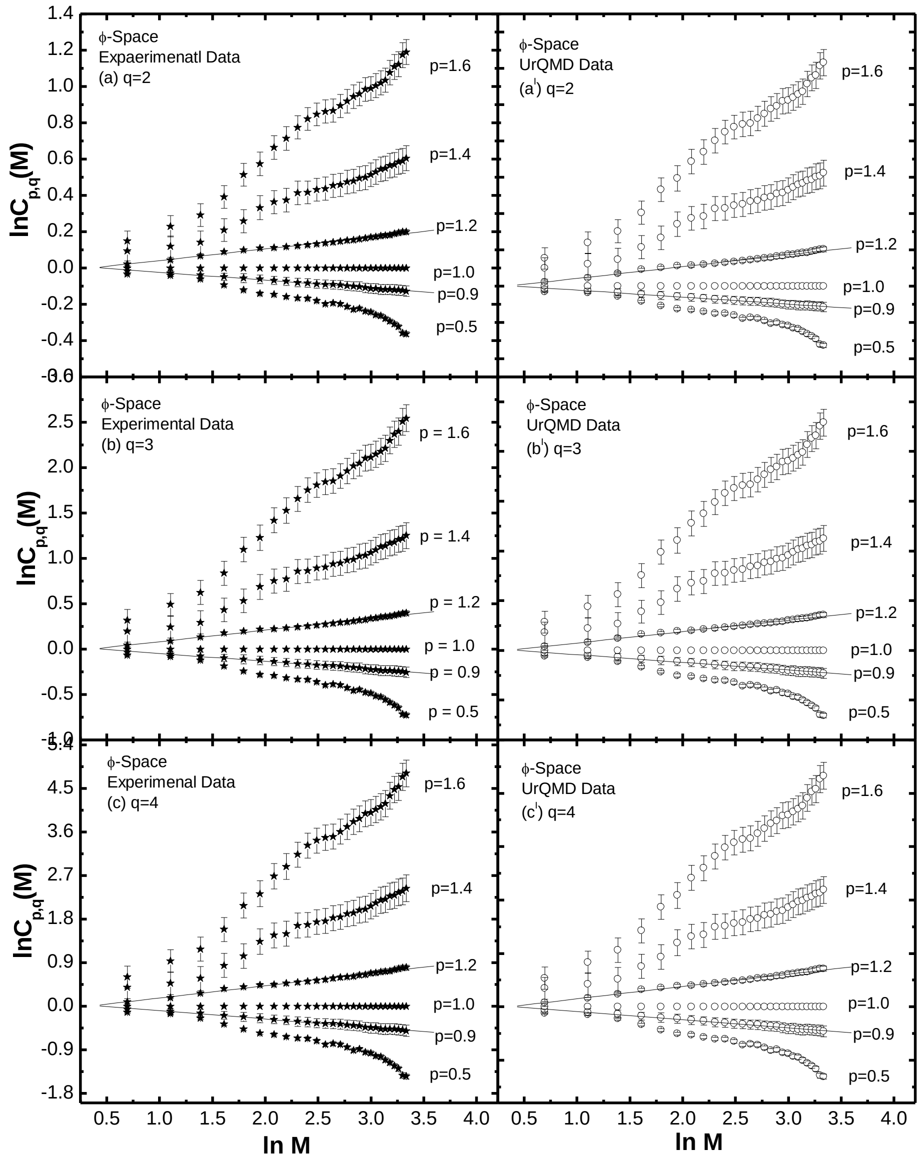


Fig-1: Variations of $\ln C_{p,q}(M)$ as function of ln M in ϕ-space (1D) in the collisions of ^{28}Si+Em at energy 14.6A GeV

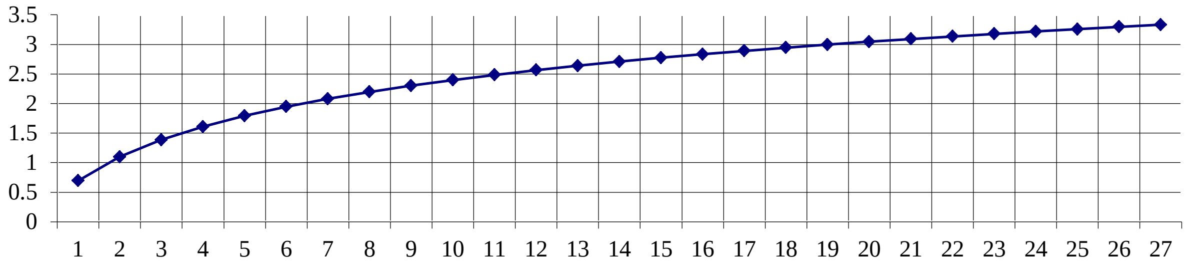


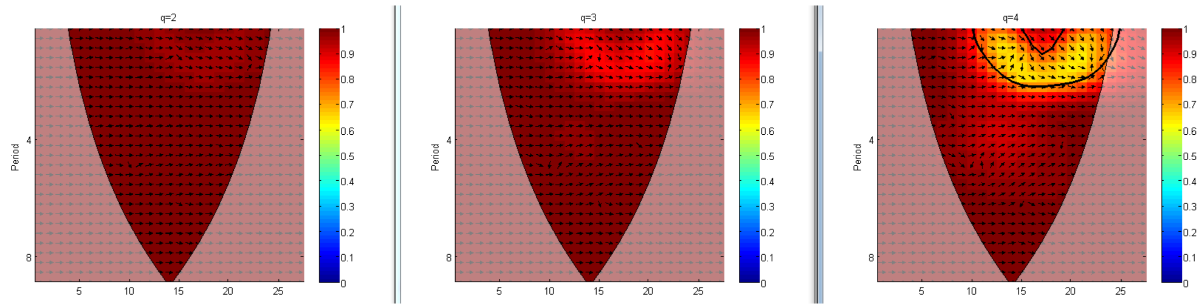
Fig-2: The correspondence between a sequence number and value of $\mathrm{Ln}(\mathrm{M})$

In doing so, we use not absolute values for normalized factorial moments but the values of the natural logarithm for normalized factorial moments. The change in the values of the natural logarithm of the parameter M is shown in Fig. 2.

In Fig. 2. along the x-axis, the order values for the parameter M are analyzed, and the values of the natural logarithm for the parameter M are plotted along the y-axis.

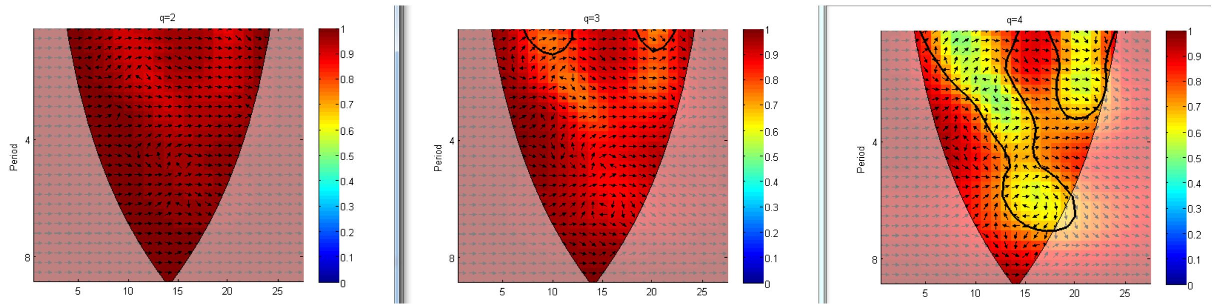
RESULTS AND DISCUSSION

Thus, we calculate the wavelet coherence for each pair of time series of the natural logarithm $C_{p,q}(M)$ (experimental data and UrQMD data) for order of moments q = 2, 3, 4 and for p = 0.5, 0.9, 1.2, 1.4 and 1.6. On Fig. 3 – Fig. 7 you can check the results of wavelet coherence between selected time series. Each of the following figures indicated a separate group of time series that match each other.



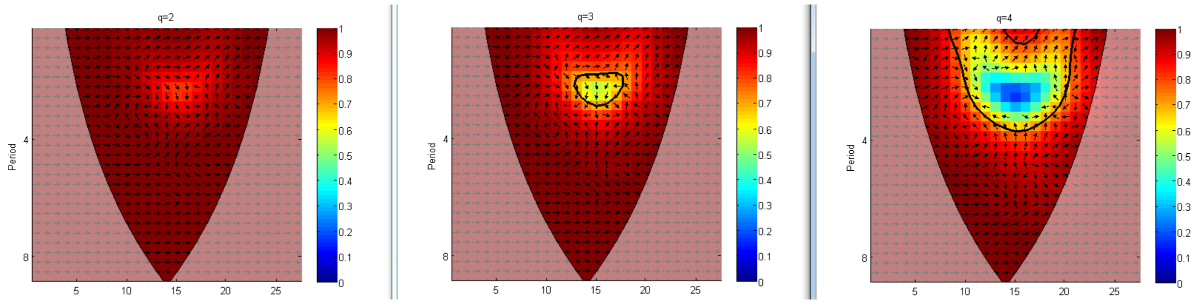
a) q=2 b) q=3 c) q=4

Fig-3: Wavelet coherence between the experimental data and UrQMD data for p=0.5



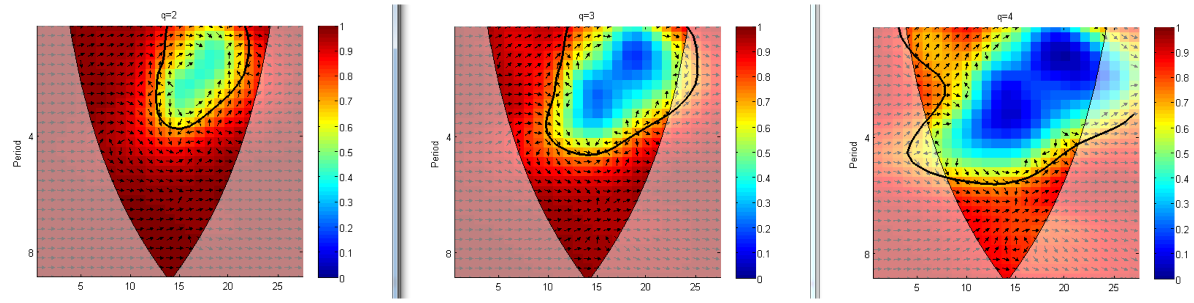
a) q=2 b) q=3 c) q=4

Fig-4: Wavelet coherence between the experimental data and UrQMD data for p=0.9



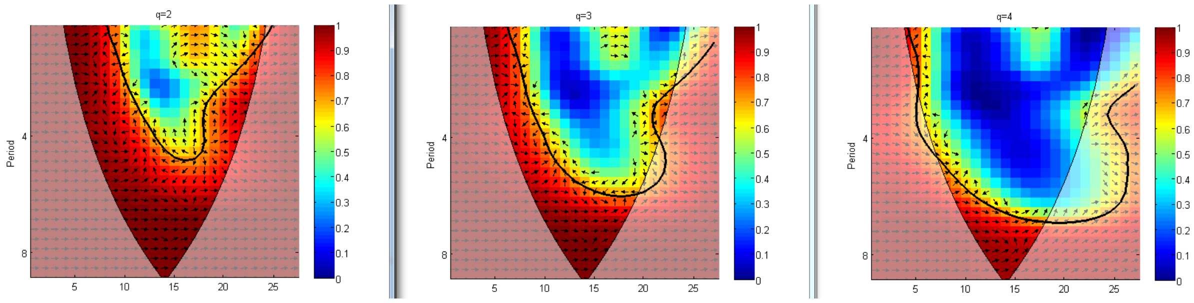
a) q=2 b) q=3 c) q=4

Fig-5: Wavelet coherence between the experimental data and UrQMD data for p=1.2



a) q=2 b) q=3 c) q=4

Fig-6: Wavelet coherence between the experimental data and UrQMD data for p=1.4



a) q=2 b) q=3 c) q=4

Fig-7: Wavelet coherence between the experimental data and UrQMD data for p=1.6

We see that as the value of the parameter p increases, the consistency between theoretical and experimental data decreases. This change is also affected by the change in the parameter q.

We can also compare the wavelet coherence for ϕ-space and η-space (see [15]). By making a comparison, we can say that the wavelet coherence for η-space is higher than for ϕ-space. But it should also be noted that the wavelet coherence for η-space and ϕ-space strongly depends on the values of the parameter p. This is due to the general dynamics of the values of the natural logarithm $C_{p,q}(M)$ (see Fig. 1).

CONCLUSIONS

In this paper, we continued our consideration of the possibility of using the ideology of wavelets to analyze the results of collisions of relativistic heavy ion. For this we use the wavelet coherence. We calculated the wavelet coherence between theoretical and experimental data for study collisions of relativistic heavy ion in ϕ-space. We have identified a different consistency between theoretical and experimental data as a function of the change in the values of the parameters p and q. There was also noted a different wavelet consistency between theoretical and experimental data for η-space and ϕ-space. This should be considered when choosing a model for the study and to evaluate the reliability of experimental data.

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