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Policy Decision Making and Management Effectiveness: Avoiding the Risk of Credibility Premium Goof

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Abstract: This article reports on estimating credibility factor using Empirical Bayes Credibility Theory. The credibility factor determined will help insurance companies to determine accurate credibility premium in order to charge aggregate premium that is reasonable in the coming year per risk using credibility premium formula. Thirteen risks/portfolios, from the Metropolitan Insurance Company, Ghana were used for the study. Among other things, the study results reveal that the distribution of all number of claims follows poisson distribution. . The study indicates that motto comprehensive, third party, workman compensation, fire material damage and asset risk had the highest credibility factors $(Z_i > 0.5)$ showing reliance on expected individual aggregate claims $\bar{X}_i$ and those with credibility factors $(Z_i < 0.5)$ indicating reliance on risk parameter of overall risk E[m(θ)] or Expected Aggregate Claims. The results further indicate that expected range of volume of business in the coming years for all risks/portfolios was estimated as $\left(V_B \pm \boldsymbol{t}_{\alpha/2}\frac{s}{\sqrt{n}}\right)$ and the empirical bayes credibility theory premium was $R_p \times \left(V_B \pm \boldsymbol{t}_{\alpha/2}\frac{s}{\sqrt{n}}\right)$.
Keywords: policy decisions, management effectiveness, credibility premium, credibility factor, Empirical Bayes credibility Theory; insolvency

INTRODUCTION

The efficiency and sustainability of insurance companies depend on their ability to disburse funds quickly and timely to their customers when claim complains are made.

Therefore, the growth and progress of any insurance company in Ghana depends on its ability to meet its expected claims emanating from the risks sold to its clients. In view of this, accurate determination of credibility premiums by Actuaries is crucial to avoiding insolvency by these companies. The parameters that lead to credibility premium determination from past data through credibility factor must be estimated using actuarial modeling.

In non-life insurance, many products or different types of risk are sold to customers, and customers transfer their risk to the insurance company by buying various risk products. The price or premium of these risks must be determined by the insurance company accurately to meet future loses and, this depends on the expected future claims. Accurate estimation of the expected future claims is essential for insurers to maintain adequate capital to pay losses and to efficiently price their insurance products. Meanwhile, many insurance companies in Ghana, in their effort to estimate future loses or claims resort to the use of simple averages and chain ladder method in their estimations.

There is, however, little information in the Ghanaian context on using empirical Bayes credibility model to determine the credibility factors for various risks. This study seeks to provide further empirical details for estimating expected future claims, credibility factors and risk premiums for some risks in Metropolitan Insurance Company, Ghana by using Empirical Bayesian's Credibility Theory Model (EBCT Model). Understanding the model will help various Insurance companies in Ghana in pricing their products.

REVIEW OF RELEVANT LITERATURE

In using a full Bayesian approach to analyze actual data on worker's compensation insurance[1], investigated two problems; first, he calculated the joint posterior distribution of the relative frequency of claims in each of the 133 rating groups. Second, he employed three distinct prior distributions and showed that the results were virtually identical in all three instances. According to [2] credibility factor shows reliance to the small data set than the related data, that is, the prior. He arrived on this conclusion when he used $C = Z\bar{x}_i + (1 - Z)\mu,$ the credibility premium to determine the credibility factor of a small insurance loss data. By estimating the credibility factor of a sample of an insurance data set [4] used credibility modeling in ratemaking process to adjust future premiums according to past experience of risk or group of risk. He considered two sets of data; the collection of current observation from the most recent period and the collection of observation from one or more prior periods. In similar studies [3] provided a Bayesian analysis to credibility theory by carefully choosing a parametric conditional loss distribution for each risk and a parametric prior from insurance loss data. He argued that, such prior must be evidence-based in some sense, be it a summary of data or an expert's opinion on the topic.

In practical application of credibility theory the structure parameter usually have to be estimated from the data [5]. This leads to an estimator of the posterior mean which is often biased especially where the credibility factor depends on the data. A more coherent approach to the problem would be to also treat the unknown parameters as random variables and simultaneously estimate the posterior mean and the structural parameters.

Therefore aim of credibility is to limit fluctuation of rate levels from year to year, that is to limit the effect that random fluctuation in the data can have on the estimate and the greatest accuracy approach to make the estimation error as small as possible [3, 6]. The most well developed approach to greatest accuracy credibility is the least square credibility, which minimizes the expected value of the squares of the estimation error Gary [3]. Further studies by [4] examined the compatibility of the Bayesian and Buhrmann models and showed that the Buhrmann credibility estimate is the best linear approximation of the Bayesian estimates of the fair premium. It was shown that the credibility estimate is equal to the Bayesian estimate for a large class of problems (exponential family/conjugate priors). Therefore, the Bayesian inference is to assess model fitting to obtain posterior predictive draws from the fitted model and compare them to the actual observed data [7]. They further indicated that hierarchical models are the powerful tool for data analysis. Hierarchical specifications are usually helpful in specifying a joint prior distribution for a set of parameters

However, in their studies [8] indicated that, the modern and more flexible approach to credibility theory concentrates on finding the most accurate estimate of an insured’s pure premium. This is accomplished by striking a balance between the individual’s risk experience and the average claim overall risk classes. They further indicated that the credibility theory model is one of the most frequently applied credibility models in insurance practices, and it enjoys some desirable optimality properties.

METHODOLOGY

The methodology used was the empirical Bayes credibility model to calculate claims per unit volumes from aggregate claims and volumes of claims. It also looks at the method of computing the credibility factors of each risk, risk per unit volume and empirical Bayesian credibility theory premiums for each risk.

Definitions of terms

Independent claims per unit volume *(X_{ij})*

Independent claims per unit volume is the ratio of the amount of claim y_{ij} to volume of claims p_{ij}. The claims for each risk and volume of risks with respect to each year must be used to compute independent claims per unit volume X_{ij}. It is denoted by,

$$X_{ij} = \frac{y_{ij}}{p_{ij}}$$

Table-1: **Claim per unit volume**

	Total Claims per Unit Volume (X_{ij})				
Risk	Year(j)				
	1	2	3	4	n
1	$\frac{y_{11}}{p_{11}}$	$\frac{y_{12}}{p_{12}}$	$\frac{y_{13}}{p_{13}}$	………	$\frac{y_{1n}}{p_{1n}}$
2	$\frac{y_{21}}{p_{21}}$	$\frac{y_{22}}{p_{22}}$	$\frac{y_{23}}{p_{23}}$	………	$\frac{y_{2n}}{p_{2n}}$
3	$\frac{y_{31}}{p_{31}}$	$\frac{y_{32}}{p_{32}}$	$\frac{y_{33}}{p_{33}}$	………..	$\frac{y_{3n}}{p_{3n}}$
4	……….	……….	………	………	………
n	$\frac{y_{n1}}{p_{n1}}$	$\frac{y_{n2}}{p_{n2}}$	$\frac{y_{n3}}{p_{n3}}$	……….	$\frac{y_{nn}}{p_{nn}}$

Risk parameter $m(\theta_i)$ of individual risk

If an unknown risk parameter θ_i is to be estimated from a sample of independent and identical random variables of claims per unit volume of risk/portfolio i, $X_{i1}, X_{i2}, \ldots\ldots\ldots X_{in}$, observed over a period of time, then the conditional claims per unit volume under a specific portfolio/risk for an insurance company given the unknown risk parameter θ_i is X_{in}/θ_i. Under Empirical Bayesian Credibility Model this unknown risk parameter θ_i is the expectation of the conditional claims per unit volume of a specific portfolio over time $[E(X_{in}/\theta_i) = \bar{x}_i = m(\theta_i)]$, but the unknown risk parameter θ_i is a function of expectation of independent conditional claims amount, denoted by:

$$m(\theta_i) = E(y_{in}/\theta_i) = \bar{x}_i$$

$$\bar{x}_i = \frac{\sum_{j=1}^{n} y_{ij}}{\bar{p}_i}$$

Where $\bar{P}_i$ is the sum of all volume of risk i in each year j and y_{ij} is the claim amount for risk i in each year j

Risk Parameter of overall risk $E[m(\theta)]$ or expected aggregate claims

This is the mean or the average of all the averages of individual risks/portfolios considered in the Empirical Bayesian Credibility Theory model. The obvious estimator of $E[m(\theta)] = \bar{x}$ is the average of the estimators of $m(\theta_i)\ for\ i = 1,2,3,\ldots n$.

$$E[m(\theta)] = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{y_{ij}}{\bar{p}}$$

$$E[m(\theta)] = \sum_{i=1}^{n} \bar{x}_i$$

Where, $\bar{P}$ is the summation of all the individual claim volumes?

$$\bar{P} = \sum_{i=1}^{n} \bar{p}_i$$

$$i = 1,2,3,4,\ldots n$$

Variability of data from year to year per Risk ($S^2(\theta_i)$)

$S^2(\theta_i)$ is the variability of the conditional claims per unit volume given an individual risk parameter. X_{in}/θ_i . $S^2(\theta_i)$ is a random variable and it is a function of an unknown parameter θ_i

$$S^2(\theta_i) = Var(X_{ij}/\theta_i)$$

$$S^2(\theta_i) = \frac{\sum_{j=1}^{n} p_{ij}(x_{ij} - \bar{x}_i)^2}{n-1}$$

Average variability of data from year to year per risk $E[\,S^2(\theta)]$

This is the mean or the average of all the variability of the conditional claims per unit volume given an individual risk considered in the Empirical Bayesian Credibility Theory model 2. The obvious estimator of $E[\,S^2(\theta)]$ is the average of the estimators of $S^2(\theta_i)\ for\ i = 1,2,3,\ldots n$.

$$E[\,S^2(\theta)] = \frac{1}{N} \sum_{i=1}^{n} \frac{1}{n-1} \sum_{j=1}^{n} p_{ij}\left(x_{ij} - \bar{x}_i\right)^2$$

Where $N\ and\ n$ are number of risks/portfolios and years considered.

Variability of the average data values for different risks $\boldsymbol{var}[\boldsymbol{m}(\boldsymbol{\theta})]$

This is the variance of all the averages of individual risks considered in the Empirical Bayesian Credibility Theory Model. The obvious estimator of $var[m(\theta)]$ is the variance of the estimators of (θ_i) $for\ i = 1,2,3, \dots n..$. The variance of all the overall means is denoted by,

$$var[m(\theta)] = \frac{1}{P^*}\left[\frac{1}{Nn-1}\sum_{i=1}^{n}\sum_{j=1}^{n} P_{ij}\left(x_{ij} - \bar{x}\right)^2 - E[\,S^2(\theta)]\right]$$

$$\text{Where } P^* = \sum_{i=1}^{n} \bar{P}_i\left(1 - \frac{\bar{P}_i}{\bar{P}}\right)$$

Estimating credibility factor using Empirical Bayesian Credibility Theory

According to [9], it is a factor which helps Actuaries' to determine the true expected claim amount in general insurance for appropriate estimation of next period or next year premium. The value of the Z shows whether next period premium estimation should depend on the population mean μ or the current sample mean $\bar{x}$, or both, where $0 < Z < 1$. Under empirical Bayesian credibility theory, the credibility factor of each risk is computed as:

$$Z_i = \frac{\sum_{j=1}^{n} P_{ij}}{\sum_{j=1}^{n} P_{ij} + \frac{E[\,S^2(\theta)]}{var[m(\theta)]}}$$

Where $i = 1,2,3, \dots n\ the\ number\ of\ risk$

Risk premium per unit volume

The implication of the credibility factor under Bayesian credibility theory is that, it helps in estimating the risk premium per unit volume, given the individual risk mean and aggregate mean. Insurance companies will be able to use the risk per unit volume computed to estimate premiums with regard to coming years' volumes of business. The risk premium per unit volume is denoted by;

Risk Premium per Unit Volume for Risk $i\ = Z_i\bar{x}_i + (1 - Z_i)E[m(\theta)]$

The next year Empirical Bayesian Credibility Theory model 2 premium for each risk is estimated by multiplying the risk premium per unit volume by the volume of the risk in that year.

Estimating future volume of business using Poisson Model

The indicators or the number of claims of various risks/portfolios at metropolitan insurance such as Asset all Risk follow Poisson distribution.

The Poisson model is defined as;

$$f(x_i/\mu) = \frac{e^{-\mu}\mu^{x_i}}{x_i!}$$

Where, $i = 0,1,2, \dots \dots \dots n$

$$E(x_i) = \mu$$

Where x_i is a random number of claims for each risk from year to year.

The likelihood function

The maximum likelihood method was applied to estimate the expected number of claims for all the risks/portfolios at metropolitan insurance.

The likelihood function of Poisson model in estimating the parameter μ

$$l(\mu) = \prod_{i=1}^{n} \frac{e^{-\mu}\mu^{x_i}}{x_i!}$$

$$l(\mu) = \frac{e^{-n\mu}\mu^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i!}$$

The log likelihood of the likelihood function was then determined for the parameter estimates

$$\ln\big(l(\mu)\big) = -n\mu + \sum_{i=1}^{n} x_i\, ln\mu - ln\prod_{i=1}^{n} x_i!$$

$$\frac{\delta}{\delta\mu}\log\big(l(\mu)\big) = -n + \frac{1}{\mu}\sum_{i=1}^{n} x_i$$

$$-n + \frac{1}{\mu}\sum_{i=1}^{n} x_i = 0$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Under Poisson model, the expectations of the individual number of risk for the coming years are equal to the parameter estimateμ.

RESULTS AND DISCUSSIONS

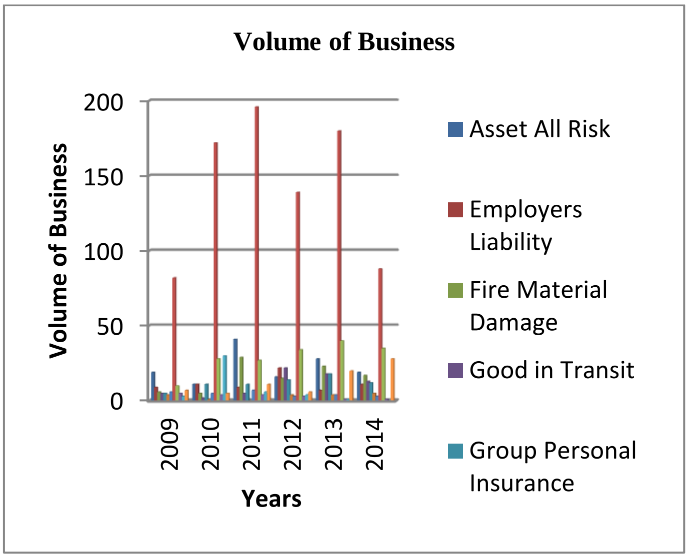

Fig-1: **volume of business for twelve risks**(P_{ij}) , $P_{ij} < 200$

Figure 1 above represents the graph of volume of business or number of claims of twelve various portfolios$\big(P_{ij} < 200\ per\ year\big)$ in table 1 appendix A in the insurance company from 2009 to 2014. The number of risk loses varies among the portfolios as depicted in the diagram above. It clearly indicates that volume of business shows significant increase in the time period. Among the claims, third party recorded the highest number of claims $P_{third\ party} > 50\ per\ year$ throughout the period of study and the rest of the portfolios show marginal increase from time to time$(P_{rest} < 50\ pe\ year)$. The range of number of claims for all the twelve risks at metropolitan insurance company lies between$0 < P_{ij} < 200$. Figure 2 below represents the graph of volume of business or number of claims of motto comprehensive $\big(\mathbf{200 < P_{motto\ comp.} < 3000\ per\ year}\big)$ in the insurance company from 2009 to 2014. It clearly indicates that volume of business for motto comprehensive insurance shows a steady increase in the time period with the least claim $P_{ij} > 1000\ per\ year$.

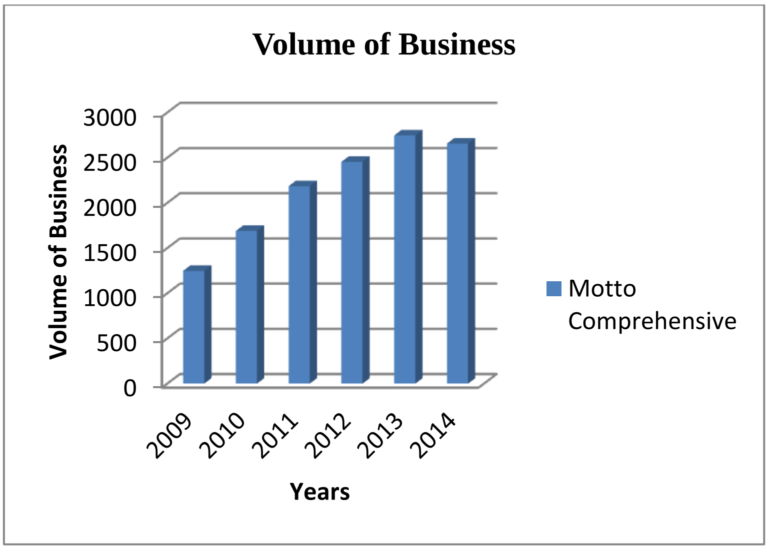

Fig-2: **Volume of business for motto comprehensive** (P_{ij}) $200 < P_{ij} < 3000$

Estimating claims per unit volume (X_{ij})

Claim per unit volume is the ratio of amount of claim y_{ij} to volume of claims p_{ij} for various risks from Metropolitan insurance Company for the years 2009 to 2014. The amount of claim and volume of claims are shown in Appendix A and B. The claims per unit volume for each risk were used in determining the average variability of data from year to year per risk $E[\,S^2(\theta)]$, variability of the average data values for different risks $var[m(\theta)]$, credibility factor (Z) and risk premium per unit volume(R_p). Table 1 below shows the total claims per unit volume of Metropolitan Insurance Company for each risk.

Table-1: **Claim per unit volume (X_{ij})**

Portfolios/ Risk(i)	Total Claim Amount per unit volume(Portfolios) X_{ij} Year(j)					
	2009	2010	2011	2012	2013	2014
Asset All Risk	15875.61	15,922.60	44,271.38	55,876.28	151,707.94	375227.87
Employers Liability	2033.78	4693.36	3591.76	4154.58	12837.85	9632.16
Fire Material Damage	6934.57	13467.48	25037.34	5722.46	44633.69	59886.12
Good in Transit	713.76	4856.89	22094.82	26337.77	25120.70	40977.65
Group Personal Insurance	5766.92	2495.07	2248.54	1550.33	11266.36	5885.05
Money Insurance	17161.18	6,506.60	0	12882.93	8314.39	13960.68
Motto Comprehensive	2578.97	3128.71	3961.79	3845.57	5181.30	6404.26
Public Liability	423.36	530.12	1442.19	6594.86	51372.90	16743.63
Third Party	2443.26	1472.97	2585.49	2705.47	2378.82	3015.00
Workman Compensation	2986.15	2924.61	4702.95	5522.07	4427.93	7495.03
Contractors All Risk	4226.00	58873.05	14789.92	27763.56	59,410.89	15,597.16
Marine Cargo	17987.72	12735.51	25042.94	73676.22	31,054.19	0
Met Executive Motto scheme	6064.59	476.78	21783.61	2416.33	3375.46	4118.78

Source: Metropolitan Insurance Company

Risk parameter of overall risk $\mathbf{E[m(\theta)]}$ **or expected aggregate claims**

This is the mean or the average of all the averages of individual risks/portfolios considered in the Empirical Bayesian Credibility Theory model. The obvious estimator of $E[m(\theta)] = \bar{x}$ is the average of the estimators of $m(\theta_i)\ for\ i = 1,2,3,\dots n.$ $\bar{P}_i, \bar{P}\ and\ P^*$, estimated from Appendix B for determining $E[m(\theta)] = \bar{x}$. $\bar{P}_i = \bar{P}_{(Asset\ All\ Risk)}$ was obtained by summing all the volume of business for Asset All Risk $\bar{P}_{(Asset\ All\ Risk)} = \sum_{i=1}^{6} P_{(Asset\ All\ Risk)} = [19 + 11 + 41 + 16 + 28 + 19] = 134.$ $\bar{P} = [\bar{P}_{(Asset\ All\ Risk)} + \bar{P}_{(Employers\ Liability)} + \cdots + \bar{P}_{(Met.Executive\ )}]$

$$= 14630.$$

$$P^* = \frac{1}{Nn-1}\sum_{i=1}^{n} \bar{P}_i\left(\mathbf{1} - \frac{\bar{\boldsymbol{P}}_i}{\bar{\boldsymbol{P}}}\right) = \frac{1}{(13\times 6)-1}[132.77 + 68.67 + \cdots + 76.59] = 39.74$$

Table-2: Determinants of $\bar{P}_i$, $\bar{P}$ and P^*

Risks/Portfolios	$\bar{\boldsymbol{P}}_i = \sum_{i=1}^{n} \boldsymbol{P}_i$	$\bar{\boldsymbol{P}}_i\left(\mathbf{1} - \frac{\bar{\boldsymbol{P}}_i}{\bar{\boldsymbol{P}}}\right)$
Asset All Risk(AAR)	134	132.77
Employers Liability(EL)	69	68.67
Fire Material Damage	95	94.38
Good in Transit	65	64.71
Group Personal Insurance	71	70.65
Money Insurance	18	17.97
Motto Comprehensive	12979	1464.68
Public Liability	28	27.95
Third Party	857	806.79
Workman Compensation	174	171.93
Contractors All Risk	18	17.97
Marine Cargo	45	44.86
Met Executive Motto scheme(MEMS)	77	76.59
	$\bar{P} = 14630$	$P^* = 39.74$

Source: Metropolitan Insurance Company

Determinants of $E[S^2(\theta)]$ $var[m(\theta)]$, credibility factor and risk premium

The risk parameter $E[m(\theta)] = \bar{X} = \sum_{j=1}^{n}\left(\frac{\sum_{j=1}^{n} y_{ij}}{\bar{P}}\right)$ is the overall mean estimate from all the thirteen risks considered in Metropolitan Insurance Company for this paper. It is used in estimating the parameters leading to credibility estimates. The overall mean was computed from appendix A of claim amount of various risks for each year. The risk parameter was computed as,

$$E[m(\theta)] = \frac{1}{14630}[301{,}636.68 + 175{,}148.6 + 1{,}815{,}126.47 + \cdots + 67{,}509.25 + 115{,}326.04]$$
$$5680.29$$

The mean for each risk or portfolio was obtained from the claim amount from Metropolitan insurance company as shown in Appendix A. The mean for Asset All Risk is shown below.

$$\bar{\boldsymbol{X}}_{Asset\ all\ Risk} = \frac{\sum_i^n y_{ij}}{\bar{P}_i} = \frac{1}{134}(301{,}636.6 + 175{,}148.64 + \cdots + 7{,}129{,}329.56) \quad = 108679.73$$

The means for each risk were obtained in order to be able to calculate the following parameters $\sum P_{ij}\left(X_{ij} - \bar{X}_i\right)^2$, and $\sum P_{ij}\left(X_{ij} - \bar{X}\right)^2$ for estimating average variability of data from year to year per risk $E[S^2(\theta)]$ and variability of the average data values for different risks $var[m(\theta)]$ as shown in table 3 below.

$$\sum \boldsymbol{P}_{ij}\left(\boldsymbol{X}_{ij} - \bar{\boldsymbol{X}}_i\right)^2 = [(15875.61 - 108679.73)^2 \times 19 + \cdots + (375227.87 - 108679.73)^2 \times 19]$$
$$+$$
$$[(2033.78 - 5644.59)^2 \times \mathbf{9} + \cdots + (9632.16 - 5644.59)^2 \times 11]$$
$$\cdots \cdots \cdots$$
$$\cdots \cdots \cdots$$
$$+$$
$$[(6064.59 - 4456.21)^2 \times 5 + \cdots + (4118.78 - 4456.21)^2 \times \mathbf{1}]$$
$$= 2.021 \times 10^{12}$$

$$\sum \boldsymbol{P}_{ij}\left(\boldsymbol{X}_{ij} - \bar{\boldsymbol{X}}\right)^2 = [(15875.61 - 5680.29)^2 \times 19 + \cdots + (375227.87 - 5680.29)^2 \times 19]$$

$$+$$
$$[(2033.78 - 5680.29)^2 \times \mathbf{9} + \cdots + (9632.16 - 5680.29)^2 \times 11]$$
$$\dots\dots\dots..$$
$$\dots\dots\dots..$$
$$+$$
$$[(6064.59 - 5680.29)^2 \times 5 + \cdots + (4118.78 - 5680.29)^2 \times 1]$$
$$= 3.474 \times 10^{12}$$

Table-3: Determinants of $E[S^2(\theta)]$ and $var[m(\theta)]$

Risks/Portfolios	$\bar{X}_i = \frac{\sum_i^n y_{ij}}{\bar{P}_i}$	$\sum_{j=1}^{n}\left(\frac{\sum_{j=1}^{n} y_{ij}}{\bar{P}}\right)$	$\sum P_{ij}(X_{ij} - \bar{X}_i)^2$	$\sum P_{ij}(X_{ij} - \bar{X})^2$
Asset All Risk	108679.73	995.42	187.47×10^{10}	3.30×10^{12}
Employers Liability	5644.59	26.62	751173403.6	751261373.4
Fire Material Damage	31215.83	202.70	3.408×10^{10}	3.60×10^{10}
Good in Transit	25970.30	115.38	7099943671	3.39×10^{10}
Group Personal Insurance	5294.66	25.69	1217745534	1042155702
Money Insurance	12763.55	15.70	202908728.1	1037442402
Motto Comprehensive	4456.21	3953.32	2.035×10^{10}	3.98×10^{10}
Public Liability	10385.46	19.87	5.660×10^{10}	9145123799
Third Party	2368.52	138.74	200209541.4	9598303785
Workman Compensation	4976.57	59.18	403684316.3	489851959
Contractors All Risk	26337.81	32.40	8427531600	1.611×10^{10}
Marine Cargo	20267.67	62.34	1.34×10^{10}	2.333×10^{10}
Met Executive Motto scheme	6257.00	32.93	3330680637	3227340665
		$\bar{X} = 5680.29$	2.021×10^{12}	3.474×10^{12}

Source: Metropolitan Insurance Company.

Table 3 above contains the determinants, $\sum_{j=1}^{n}\left(\frac{\sum_{j=1}^{n} y_{ij}}{\bar{P}}\right)$, $\sum \boldsymbol{P_{ij}}(\boldsymbol{X_{ij}} - \bar{\boldsymbol{X}}_i)^2$ and $\sum \boldsymbol{P_{ij}}(\boldsymbol{X_{ij}} - \bar{\boldsymbol{X}})^2$ of $E[S^2(\theta)]$, $var[m(\theta)]$ and $\bar{X}$. The average variability of data from year to year per risk and variability of the average data values for different risks were used in determining the credibility factor, risk premium and empirical Bayes credibility theory premium.

Average variability of data from year to year per risk $E[S^2(\theta)]$

This is the mean or the average of all the variability of the conditional claims per unit volume given an individual risk considered in the Empirical Bayesian Credibility Theory model. The obvious estimator of $E[S^2(\theta)]$ is the average of the estimators of $S^2(\theta_i)$ $for\ i = 1,2,3, \dots n.$

$$E[S^2(\theta)] = \frac{1}{N}\sum_{i=1}^{n}\frac{1}{n-1}\sum_{j=1}^{n} p_{ij}(x_{ij} - \bar{x}_i)^2$$
$$E[S^2(\theta)] = \frac{1}{13} \times \frac{1}{5} \times 2.021 \times 10^{12} = 3.109 \times 10^{10}$$

Variability of the average data values for different risks $var[m(\theta)]$

This is the variance of all the averages of individual risks considered in the Empirical Bayesian Credibility Theory model. The obvious estimator of $var[m(\theta)]$ is the variance of the estimators of (θ_i) $for\ i = 1,2,3, \dots n.$. The variance of all the overall means is denoted by,

$$var[m(\theta)] = \frac{1}{P^*}\left[\frac{1}{Nn-1}\sum_{i=1}^{n}\sum_{j=1}^{n} P_{ij}(x_{ij} - \bar{x})^2 - E[S^2(\theta)]\right]$$
$$var[m(\theta)] = \frac{1}{39.74}\left[\frac{1}{13 \times 6 - 1}(3.474 \times 10^{12}) - 3.109 \times 10^{10}\right]$$
$$352966359.3$$

Expected future volumes of risk for Metropolitan Insurance Company

Poisson distribution was used to estimate future volume of business for Metropolitan Insurance Company. The past volume of business for each risk in Appendix A from 2009 to 2014 was run under one sample t-test to access the fit of poison distribution to the past volume of business to estimate the expected future volume of business in the coming years. The test statistics are shown below.

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of Asset_all_Risk is Poisson with mean 22.	One-Sample Kolmogorov-Smirnov Test	.337	Retain the null hypothesis.
2	The distribution of Employers_Liability is Poisson with mean 12.	One-Sample Kolmogorov-Smirnov Test	.597	Retain the null hypothesis.
3	The distribution of Fire_Material_Damage is Poisson with mean 16.	One-Sample Kolmogorov-Smirnov Test	.535	Retain the null hypothesis.
4	The distribution of Goods_in_Transit is Poisson with mean 11.	One-Sample Kolmogorov-Smirnov Test	.160	Retain the null hypothesis.
5	The distribution of Group_Personal_Accident is Poisson with mean 12.	One-Sample Kolmogorov-Smirnov Test	.972	Retain the null hypothesis.
6	The distribution of Money_Insurance is Poisson with mean 3.	One-Sample Kolmogorov-Smirnov Test	.595	Retain the null hypothesis.
7	The distribution of Mottor_Comprehensive is Poisson with mean 2,163.	One-Sample Kolmogorov-Smirnov Test	.100	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of Public_Liability is Poisson with mean 5.	One-Sample Kolmogorov-Smirnov Test	.999	Retain the null hypothesis.
2	The distribution of Third_Party is Poisson with mean 143.	One-Sample Kolmogorov-Smirnov Test	.112	Retain the null hypothesis.
3	The distribution of Workman_Compensation is Poisson with mean 29.	One-Sample Kolmogorov-Smirnov Test	.848	Retain the null hypothesis.
4	The distribution of Contractors_All_Risk is Poisson with mean 3.	One-Sample Kolmogorov-Smirnov Test	.999	Retain the null hypothesis.
5	The distribution of Marine_Cargo is Poisson with mean 7.	One-Sample Kolmogorov-Smirnov Test	.076	Retain the null hypothesis.
6	The distribution of Mett_Exe_Mottor_Scheme is Poisson with mean 13.	One-Sample Kolmogorov-Smirnov Test	.193	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

The expected future volume of business of metropolitan insurance is shown in the test statistic above. The mean of Poison distribution of the various risk/portfolio for the six years $(n = 6)$ are used as the future volume of business because the past number of claims follows poison distribution with their confidence interval $V_B \pm \boldsymbol{t}_{\alpha/2}\frac{s}{\sqrt{n}}$ after normal approximations. The $t - distribution$ was used in computing the confidence interval for all the portfolios due to unknown population standard deviations. Table 4 below shows the expected number of claims of various risks indicated by the poison model in the test statistics and their $t\ distributions$.

Table-4: Expected future volume of business$[(V_B) = (\mu)]$

Risks/Portfolios	Poison Parameter $\lambda = (V_B)$	Normal Approx. Parameter		$\pm t_{\alpha/2}\frac{s}{\sqrt{n}}$
		$\lambda = \mu$	$s = \sqrt{\lambda}$	
Asset All Risk(AAR)	22	22	4.69	$\pm$4.92
Employers Liability(EL)	12	12	3.46	$\pm$3.63
Fire Material Damage	10	10	3.16	$\pm$3.32
Good in Transit	11	11	3.32	$\pm$3.48
Group Personal Insurance	12	12	3.46	$\pm$3.63
Money Insurance	3	3	1.73	$\pm$1.82
Motto Comprehensive	2163	2163	46.51	$\pm$48.82
Public Liability	6	6	2.44	$\pm$2.56
Third Party	143	143	11.96	$\pm$12.55
Workman Compensation	20	20	4.47	$\pm$4.69
Contractors All Risk	3	3	1.73	$\pm$1.82
Marine Cargo	7	7	2.64	$\pm$2.76
Met Executive Motto scheme	13	13	3.61	$\pm$3.78

Source: Metropolitan Insurance Company

Estimating credibility factor/risk premium per unit volume/EBCT premium

The credibility factors of various risks or portfolios in table 5 was determined by using the total claims of individual risk in table 1, variability of the average data values for different risks $var[m(\theta)]$ and average variability of data from year to year per risk $E[S^2(\theta)]$.

$$Z_i = \frac{\sum_{j=1}^{n} P_{ij}}{\sum_{j=1}^{n} P_{ij} + \frac{E[S^2(\theta)]}{var[m(\theta)]}}$$

The credibility factor helps in estimating the risk premium per unit volume for the Metropolitan Insurance Company, given the individual risk mean and aggregate mean. Insurance companies will be able to use the risk per unit volume computed to estimate premiums with regard to coming year's volumes of business. The risk premium per unit volume is denoted as;

Risk Premium per unit volume for $risk_i = Z_i\bar{x}_i + (1 - Z_i)E[m(\theta)]$.

The Empirical Bayesian Credibility Theory model (EBCT) premium for each risk is estimated by multiplying the risk premium per unit volume by the expected future volume of the risk in the coming year.

Table-5: credibility factor(Z)/risk premium(R_p)/ EBCT premium$(V_B \times R_p)$

Risks/Portfolios	Credibility Factor (Z)	Risk Premium per unit volume (R_p)	EBCT premium $R_p \times \left(V_B \pm t_{\alpha/2}\frac{s}{\sqrt{n}}\right) Ghs$
Asset All Risk	0.6033806	67828.154	67828.154(22 $\pm$ 4.92)
Employers Liability	0.4392608	5664.608	5664.608(12 $\pm$ 3.63)
Fire Material Damage	0.5188930	18930.503	18930.503(10 $\pm$ 3.32)
Good in Transit	0.4246088	14295.607	14295.607(11 $\pm$ 3.48)
Group Personal Insurance	0.4463105	5508.179	5508.607(12 $\pm$ 3.63)
Money Insurance	0.1696799	6882.176	6882.176(3 $\pm$ 1.82)
Motto Comprehensive	0.9932592	4464.461	4464.461(2163 $\pm$ 48.52)
Public Liability	0.2412086	6815.217	6815.217(6 $\pm$ 2.52)
Third Party	0.9067995	2677.178	2677.178(143 $\pm$ 12.55)
Workman Compensation	0.6639142	5213.080	5213.08(20 $\pm$ 4.47)
Contractors All Risk	0.1696799	9185.456	9185.456(3 $\pm$ 1.82)
Marine Cargo	0.3381372	10612.825	10612.825(7 $\pm$ 2.76)
Met Executive Motto scheme	0.4664347	5949.287	5949.287(13 $\pm$ 3.78)

Source: computed by Author

CONCLUSION

The Empirical Bayes Credibility Theory was used to model the risk rating in metropolitan insurance company in Ghana. The model will help the Metropolitan Insurance Company in determining the expected future claim amount for thirteen portfolios. The study indicates that motto comprehensive, third party, workman compensation, fire material damage and asset all risk among the thirteen portfolios have the highest credibility factors $(Z_i > 0.5)$. This shows that with respect to the above portfolios, the insurance company can place much reliance on their individual expected aggregate claims $\bar{X}_i$ for estimating their risk premium per unit volume (R_p) and EBCT premium $(V_B \times R_p)$. The rest of the portfolios had the lowest credibility factors $(Z_i < 0.5)$, indicating that metropolitan insurance company cannot rely on their individual expected aggregate claims $\bar{X}_i$ for estimating their risk premium unless they employ the overall aggregate claims, the risk parameter of overall risk E[m(θ)] or expected aggregate claims. The sturdy also indicates that the distribution of volume of business in the company follows poison distribution. It further indicates that expected range of volume of business in the coming years for all risks/portfolios was estimated as $R_p \times \left(V_B \pm \boldsymbol{t}_{\alpha/2} \frac{s}{\sqrt{n}}\right)$ in table 4.

REFERENCES

1. Klugman, S. A. (1987). Credibility for classification ratemaking via the hierarchical normal linear model. In *Proceeding of the casualty Actuarial Society LXXIV* (pp. 272–321).
2. Bailey, A. (1950). Credibility Procedures. In *Proceeding of the casualty Actuarial Society 37* (pp. 7–23, 94–115).
3. Klugman, S. A. (1992). *Bayesian Statistics in Actuarial Science with Emphasis on Credibility*. Boston Kluwar.
4. Herzog, T. N. (1994). *Introduction to Credibility Theory*. Winstead CT: ACTEX 1994.
5. Malhar Howard, C. (1990). An example of credibility and shifting risk parameters. *PCAS CXXVII*, 225–308.
6. King, G., Tomz, M., & Wittenberg, J. (2000). Making the most of statistical analyses: Improving interpretation and presentation. *American journal of political science*, 347-361.
7. Gelman, A., Carlin, J., Stern, H., & Rubin, D. (2004). *Bayesian Data Analysis*. Chapman and Hall, Boca Raton.
8. Panjer, H. H., & Wilmot, G. E. (2012). *Loss Models: From Data to Decisions*. Noboken, New Jersey: John Wiley and Sons.
9. Longley-Cook, L. H. (1962). An introduction to credibility theory. In Proceeding of the casualty Actuarial Society 49 (pp. 194–221). New York, NY 10019: Casualty Actuarial Society, 250 West 34th St. New York, NY.

APPENDICES

Appendix A Volume of business

Portfolios/ Risk(i)	Volume of Business(Portfolios) P_{ij} Year(j)					
	2009	2010	2011	2012	2013	2014
Asset All Risk	19	11	41	16	28	19
Employers Liability	9	11	9	22	7	11
Fire Material Damage	6	5	29	15	23	17
Good in Transit	5	2	5	22	18	13
Group Personal Insurance	5	11	11	14	18	12
Money Insurance	4	1	0	4	4	5
Motto Comprehensive	1248	1690	2185	2454	2746	2656
Public Liability	6	5	7	3	4	3
Third Party	82	172	196	139	180	88
Workman Compensation	10	28	27	34	40	35
Contractors All Risk	5	4	4	3	1	1
Marine Cargo	3	30	6	4	1	0
Met Executive Motto scheme	7	5	11	6	20	28

Source: Metropolitan Insurance Company

Appendix B Total claim amount

Portfolios/ Risk(i)	Total Claim Amount(Portfolios) y_{ij} Year(j)					
	2009	2010	2011	2012	2013	2014
Asset All Risk	301,636.68	175,148.64	1,815,126.47	894,020.43	4,247,822.58	7,129,329.56
Employers Liability	18,304.05	51,627.06	32,325.84	91,400.86	89,864.98	105,953.77
Fire Material Damage	41,607.45	67,337.42	726,082.97	85,837.03	1,026,574.97	1,018,064.06
Good in Transit	,568.813	9,713.79	110,474.14	579,430.99	452,172.68	532,709.52
Group Personal Insurance	28,834.64	27,445.81	24,734.03	21,704.64	202,794.45	70,620.60
Money Insurance	68,644.71	6,506.60	0	51,531.70	33,257.57	69,803.44
Motto Comprehensive	3,218,561.04	5,287,521.51	8,656,522.00	9,437,038.62	14,227,853.39	17,009,740.26
Public Liability	2,540.13	2,650.60	10,095.30	19,784.58	205,491.6	50,230.90
Third Party	200,347.56	253,351.10	506,561.84	376,060.39	428,188.39	265,320.09
Workman Compensation	29,861.54	81,889.08	126,979.84	187,750.48	177,117.25	262,325.88
Contractors All Risk	21,130.00	235,492.20	59,159.68	83,290.69	59,410.89	15,597.16
Marine Cargo	53,963.18	382,065.34	150,257.64	294,704.88	31,054.19	0
Met Executive Motto scheme	42,452.14	2,383.91	239,619.73	14,497.99	67,509.25	115,326.04

Source: Metropolitan Insurance Company